

H**ANSWERS TO ODD-NUMBERED EXERCISES****CHAPTER I****EXERCISES 1.1 ■ PAGE 20**

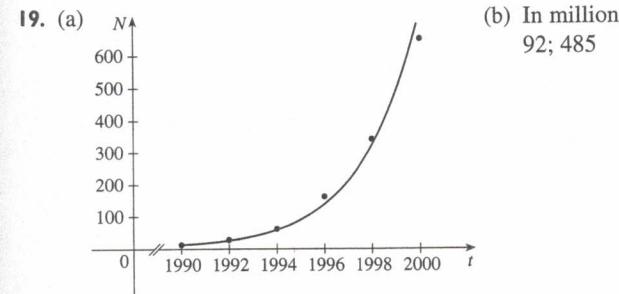
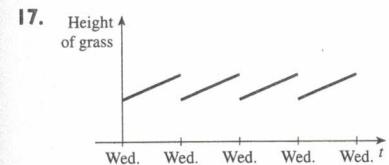
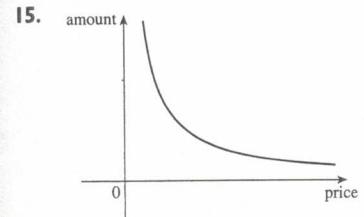
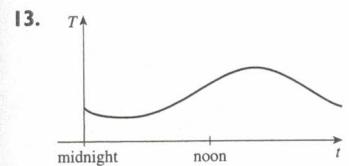
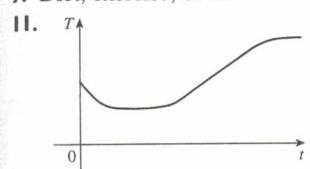
1. (a) -2 (b) 2.8 (c) -3, 1 (d) -2.5, 0.3

(e) $[-3, 3], [-2, 3]$ (f) $[-1, 3]$

3. $[-85, 115]$ 5. No

7. Yes, $[-3, 2], [-3, -2] \cup [-1, 3]$

9. Diet, exercise, or illness



21. $12, 16, 3a^2 - a + 2, 3a^2 + a + 2, 3a^2 + 5a + 4,$
 $6a^2 - 2a + 4, 12a^2 - 2a + 2, 3a^4 - a^2 + 2,$

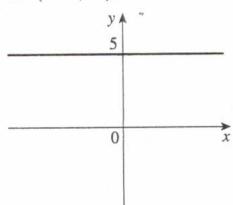
$9a^4 - 6a^3 + 13a^2 - 4a + 4, 3a^2 + 6ah + 3h^2 - a - h + 2$

23. $-3 - h$ 25. $-1/(ax)$

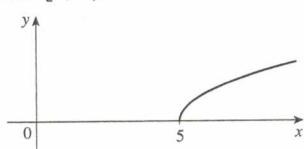
27. $\{x \mid x \neq \frac{1}{3}\} = (-\infty, \frac{1}{3}) \cup (\frac{1}{3}, \infty)$

29. $[0, \infty)$ 31. $(-\infty, 0) \cup (5, \infty)$

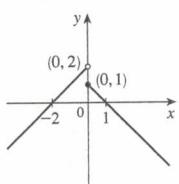
33. $(-\infty, \infty)$



37. $[5, \infty)$



41. $(-\infty, \infty)$



45. $f(x) = \frac{5}{2}x - \frac{11}{2}, 1 \leq x \leq 5$

47. $f(x) = 1 - \sqrt{-x}$

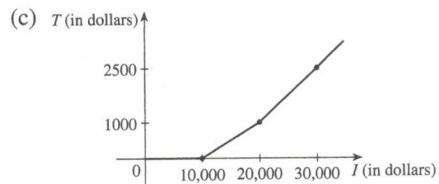
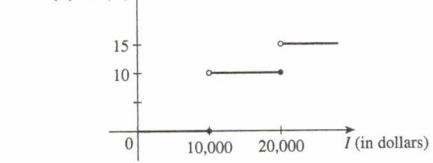
49. $f(x) = \begin{cases} -x + 3 & \text{if } 0 \leq x \leq 3 \\ 2x - 6 & \text{if } 3 < x \leq 5 \end{cases}$

51. $A(L) = 10L - L^2, 0 < L < 10$

53. $A(x) = \sqrt{3}x^2/4, x > 0$ 55. $S(x) = x^2 + (8/x), x > 0$

57. $V(x) = 4x^3 - 64x^2 + 240x, 0 < x < 6$

59. (a) R (%) 59. (b) \$400, \$1900

61. f is odd, g is even

63. (a) $(-5, 3)$ (b) $(-5, -3)$

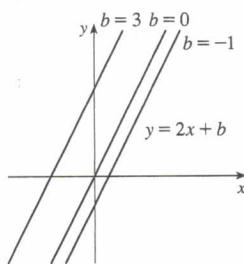
65. Odd 67. Neither 69. Even

EXERCISES 1.2 ■ PAGE 34

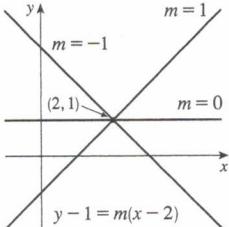
1. (a) Root (b) Algebraic (c) Polynomial (degree 9)
-
- (d) Rational (e) Trigonometric (f) Logarithmic

3. (a)
- h
- (b)
- f
- (c)
- g

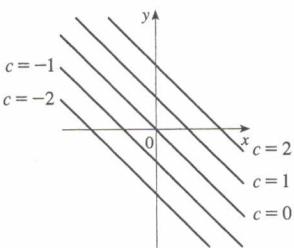
5. (a) $y = 2x + b$,
where b is the y -intercept.



- (b) $y = mx + 1 - 2m$,
where m is the slope.
See graph at right.
(c) $y = 2x - 3$

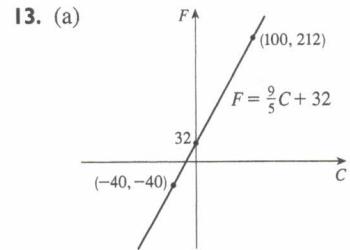


7. Their graphs have slope -1 .



9. $f(x) = -3x(x + 1)(x - 2)$

11. (a) 8.34, change in mg for every 1 year change
(b) 8.34 mg



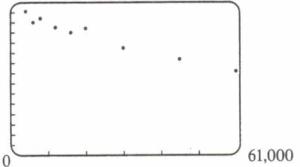
- (b) $\frac{9}{5}$, change in $^{\circ}\text{F}$ for every 1°C change; 32, Fahrenheit temperature corresponding to 0°C

15. (a) $T = \frac{1}{6}N + \frac{307}{6}$ (b) $\frac{1}{6}$, change in $^{\circ}\text{F}$ for every chirp per minute change (c) 76°F

17. (a) $P = 0.434d + 15$ (b) 196 ft

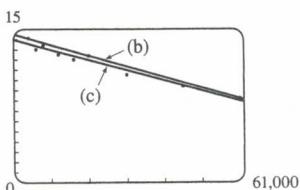
19. (a) Cosine (b) Linear

21. (a) 15



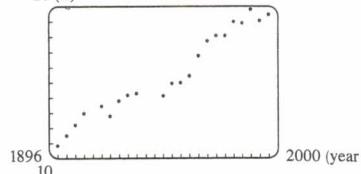
Linear model is appropriate

- (b) $y = -0.000105x + 14.521$



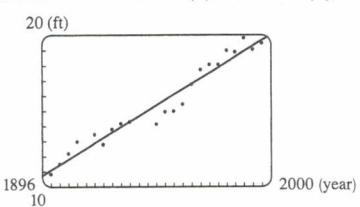
- (c) $y = -0.00009979x + 13.951$ [See graph in (b).]
(d) About 11.5 per 100 population (e) About 6% (f) No

23. (a) 20 (ft)



Linear model is appropriate

- (b) $y = 0.08912x - 158.24$ (c) 20 ft (d) No



25. $y \approx 0.0012937x^3 - 7.06142x^2 + 12,823x - 7,743,770$; 1914 million

EXERCISES 1.3 ■ PAGE 43

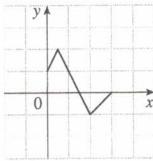
1. (a) $y = f(x) + 3$ (b) $y = f(x) - 3$ (c) $y = f(x - 3)$

- (d) $y = f(x + 3)$ (e) $y = -f(x)$ (f) $y = f(-x)$

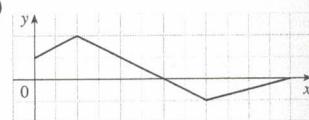
- (g) $y = 3f(x)$ (h) $y = \frac{1}{3}f(x)$

3. (a) 3 (b) 1 (c) 4 (d) 5 (e) 2

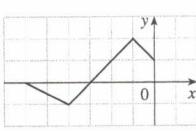
5. (a)



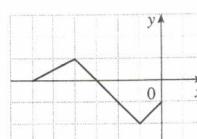
- (b)



- (c)

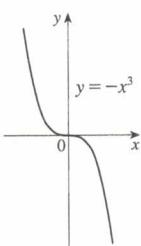


- (d)

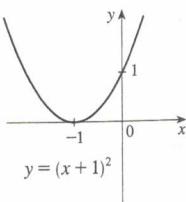


7. $y = -\sqrt{-x^2 - 5x - 4} - 1$

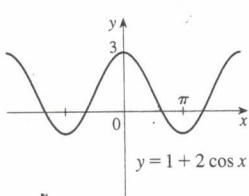
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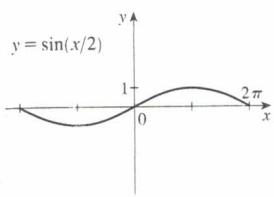
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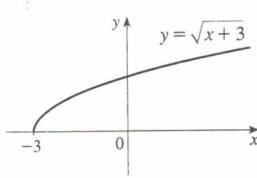
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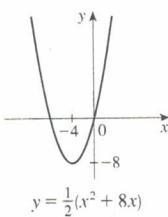
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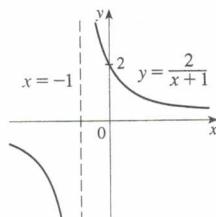
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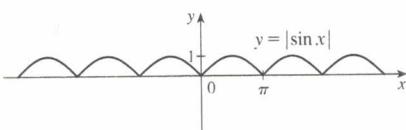
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21.



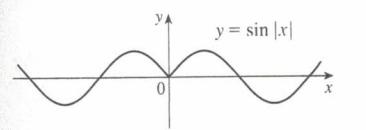
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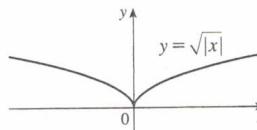
25. $L(t) = 12 + 2 \sin \left[\frac{2\pi}{365} (t - 80) \right]$

27. (a) The portion of the graph of $y = f(x)$ to the right of the y -axis is reflected about the y -axis.

(b)



(c)



29. $(f + g)(x) = x^3 + 5x^2 - 1, (-\infty, \infty)$

$(f - g)(x) = x^3 - x^2 + 1, (-\infty, \infty)$

$(fg)(x) = 3x^5 + 6x^4 - x^3 - 2x^2, (-\infty, \infty)$

$(f/g)(x) = (x^3 + 2x^2)/(3x^2 - 1), \{x | x \neq \pm 1/\sqrt{3}\}$

31. (a) $(f \circ g)(x) = 4x^2 + 4x, (-\infty, \infty)$

(b) $(g \circ f)(x) = 2x^2 - 1, (-\infty, \infty)$

(c) $(f \circ f)(x) = x^4 - 2x^2, (-\infty, \infty)$

(d) $(g \circ g)(x) = 4x + 3, (-\infty, \infty)$

33. (a) $(f \circ g)(x) = 1 - 3 \cos x, (-\infty, \infty)$

(b) $(g \circ f)(x) = \cos(1 - 3x), (-\infty, \infty)$

(c) $(f \circ f)(x) = 9x - 2, (-\infty, \infty)$

(d) $(g \circ g)(x) = \cos(\cos x), (-\infty, \infty)$

35. (a) $(f \circ g)(x) = (2x^2 + 6x + 5)/[(x + 2)(x + 1)], \{x | x \neq -2, -1\}$

(b) $(g \circ f)(x) = (x^2 + x + 1)/(x + 1)^2, \{x | x \neq -1, 0\}$

(c) $(f \circ f)(x) = (x^4 + 3x^2 + 1)/[x(x^2 + 1)], \{x | x \neq 0\}$

(d) $(g \circ g)(x) = (2x + 3)/(3x + 5), \{x | x \neq -2, -\frac{5}{3}\}$

37. $(f \circ g \circ h)(x) = 2x - 1$

39. $(f \circ g \circ h)(x) = \sqrt{x^6 + 4x^3 + 1}$

41. $g(x) = x^2 + 1, f(x) = x^{10}$

43. $g(x) = \sqrt[3]{x}, f(x) = x/(1 + x)$

45. $g(t) = \cos t, f(t) = \sqrt{t}$

47. $h(x) = x^2, g(x) = 3^x, f(x) = 1 - x$

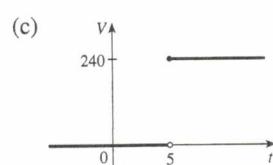
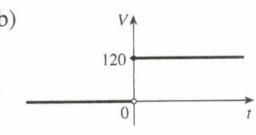
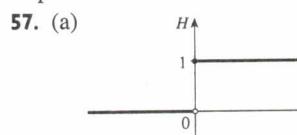
49. $h(x) = \sqrt{x}, g(x) = \sec x, f(x) = x^4$

51. (a) 4 (b) 3 (c) 0 (d) Does not exist; $f(6) = 6$ is not in the domain of g . (e) 4 (f) -2

53. (a) $r(t) = 60t$ (b) $(A \circ r)(t) = 3600\pi t^2$; the area of the circle as a function of time

55. (a) $s = \sqrt{d^2 + 36}$ (b) $d = 30t$

(c) $s = \sqrt{900t^2 + 36}$; the distance between the lighthouse and the ship as a function of the time elapsed since noon



$$V(t) = 240H(t - 5)$$

59. Yes; $m_1 m_2$

61. (a) $f(x) = x^2 + 6$ (b) $g(x) = x^2 + x - 1$

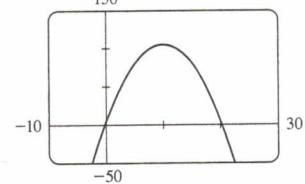
63. (a) Even; even (b) Odd; even

65. Yes

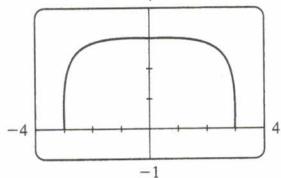
EXERCISES 1.4 ■ PAGE 51

1. (c)

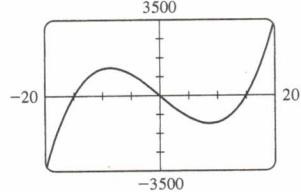
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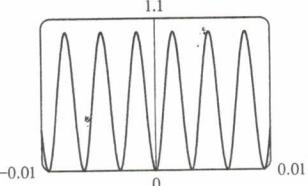
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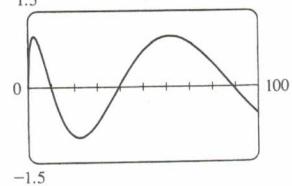
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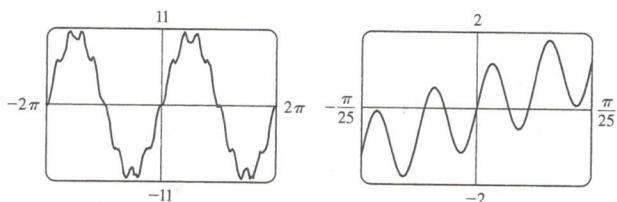
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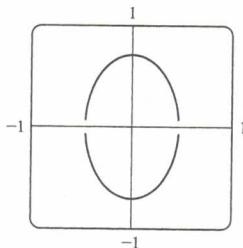
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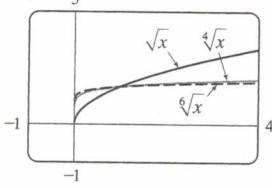
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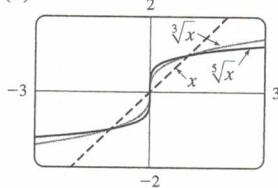
15.

17. No 19. 9.05 21. 0, 0.88 23. g 25. $-0.85 < x < 0.85$

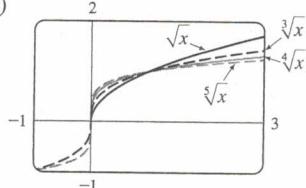
27. (a)



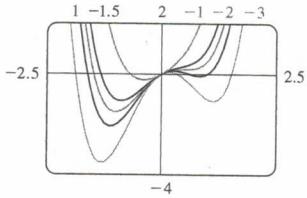
(b)



(c)

(d) Graphs of even roots are similar to \sqrt{x} , graphs of odd roots are similar to $\sqrt[3]{x}$. As n increases, the graph of $y = \sqrt[n]{x}$ becomes steeper near 0 and flatter for $x > 1$.

29.



If $c < -1.5$, the graph has three humps: two minimum points and a maximum point. These humps get flatter as c increases until at $c = -1.5$ two of the humps disappear and there is only one minimum point. This single hump then moves to the right and approaches the origin as c increases.

31. The hump gets larger and moves to the right.

33. If $c < 0$, the loop is to the right of the origin; if $c > 0$, the loop is to the left. The closer c is to 0, the larger the loop.

CHAPTER I REVIEW ■ PAGE 52

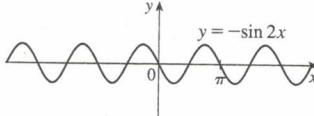
True-False Quiz

1. False 3. False 5. True

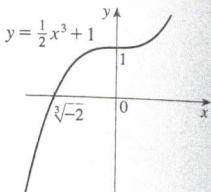
Exercises

1. (a) 2.7 (b) 2.3, 5.6 (c) $[-6, 6]$ (d) $[-4, 4]$
 (e) $[-4, 4]$ (f) Odd; its graph is symmetric about the origin.
3. $2a + h - 2$ 5. $(-\infty, \frac{1}{3}) \cup (\frac{1}{3}, \infty), (-\infty, 0) \cup (0, \infty)$
7. $\mathbb{R}, [0, 2]$
9. (a) Shift the graph 8 units upward.
 (b) Shift the graph 8 units to the left.
 (c) Stretch the graph vertically by a factor of 2, then shift it 1 unit upward.
 (d) Shift the graph 2 units to the right and 2 units downward.
 (e) Reflect the graph about the x -axis.
 (f) Reflect the graph about the line x -axis, then shift it 3 units upward.

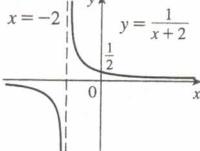
11.



13.



15.



17. (a) Neither (b) Odd (c) Even (d) Neither

19. (a) $(f \circ g)(x) = \sqrt{\sin x}, \{x | x \in [2n\pi, \pi + 2n\pi], n \text{ an integer}\}$
 (b) $(g \circ f)(x) = \sin \sqrt{x}, [0, \infty)$
 (c) $(f \circ f)(x) = \sqrt[n]{x}, [0, \infty)$
 (d) $(g \circ g)(x) = \sin(\sin x), \mathbb{R}$

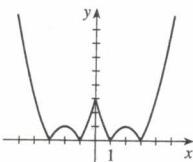
21. All have domain \mathbb{R} . The range is $[-1, 1]$ for n odd and $[0, 1]$ for n even. The spikes become sharper as $n \rightarrow \infty$.23. $y = 0.2493x - 423.4818$; about 77.6 years

PRINCIPLES OF PROBLEM SOLVING ■ PAGE 59

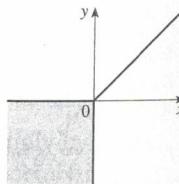
1. $a = 4\sqrt{h^2 - 16}/h$, where a is the length of the altitude and h is the length of the hypotenuse

3. $-\frac{7}{3}, 9$

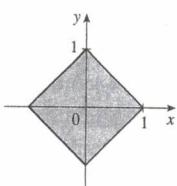
5.



7.



9.

11. 40 mi/h 15. $f_n(x) = x^{2^{n+1}}$

CHAPTER 2**EXERCISES 2.1 ■ PAGE 65**

1. (a) $-44.4, -38.8, -27.8, -22.2, -16.6$
 (b) -33.3 (c) $-33\frac{1}{3}$
 3. (a) (i) 0.333333 (ii) 0.263158 (iii) 0.251256
 (iv) 0.250125 (v) 0.2 (vi) 0.238095 (vii) 0.248756
 (viii) 0.249875 (b) $\frac{1}{4}$ (c) $y = \frac{1}{4}x + \frac{1}{4}$
 5. (a) (i) -32 ft/s (ii) -25.6 ft/s (iii) -24.8 ft/s
 (iv) -24.16 ft/s (b) -24 ft/s
 7. (a) (i) 4.65 m/s (ii) 5.6 m/s (iii) 7.55 m/s
 (iv) 7 m/s (b) 6.3 m/s
 9. (a) $0, 1.7321, -1.0847, -2.7433, 4.3301, -2.8173, 0,$
 $-2.1651, -2.6061, -5, 3.4202$; no (c) -31.4

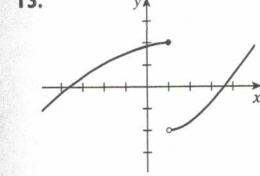
EXERCISES 2.2 ■ PAGE 74

1. Yes

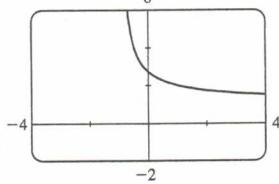
3. (a) $\lim_{x \rightarrow -3} f(x) = \infty$ means that the values of $f(x)$ can be made arbitrarily large (as large as we please) by taking x sufficiently close to -3 (but not equal to -3).
 (b) $\lim_{x \rightarrow 4^+} f(x) = -\infty$ means that the values of $f(x)$ can be made arbitrarily large negative by taking x sufficiently close to 4 through values larger than 4 .

5. (a) 2 (b) 3 (c) Does not exist (d) 4
 (e) Does not exist
 7. (a) -1 (b) -2 (c) Does not exist (d) 2 (e) 0
 (f) Does not exist (g) 1 (h) 3
 9. (a) $-\infty$ (b) ∞ (c) ∞ (d) $-\infty$ (e) ∞
 (f) $x = -7, x = -3, x = 0, x = 6$

11. (a) 1 (b) 0 (c) Does not exist



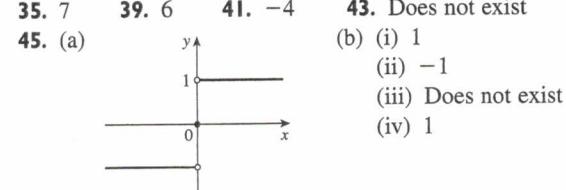
17. $\frac{2}{3}$ 19. $\frac{1}{2}$ 21. $\frac{1}{4}$ 23. $\frac{3}{5}$ 25. $-\infty$
 27. ∞ 29. $-\infty$ 31. $-\infty$ 33. $-\infty; \infty$
 35. (a) 2.71828 (b)



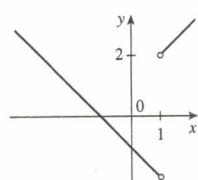
37. (a) $0.998000, 0.638259, 0.358484, 0.158680, 0.038851,$
 $0.008928, 0.001465; 0$
 (b) $0.000572, -0.000614, -0.000907, -0.000978, -0.000993,$
 $-0.001000; -0.001$
 39. No matter how many times we zoom in toward the origin, the graph appears to consist of almost-vertical lines. This indicates more and more frequent oscillations as $x \rightarrow 0$.
 41. $x \approx \pm 0.90, \pm 2.24; x = \pm \sin^{-1}(\pi/4), \pm(\pi - \sin^{-1}(\pi/4))$

EXERCISES 2.3 ■ PAGE 84

1. (a) -6 (b) -8 (c) 2 (d) -6
 (e) Does not exist (f) 0
 3. 59 5. 390 7. $\frac{1}{8}$ 9. 0 11. 5
 13. Does not exist 15. $\frac{6}{5}$ 17. 8 19. $\frac{1}{12}$ 21. 6
 23. $\frac{1}{6}$ 25. $-\frac{1}{16}$ 27. $\frac{1}{128}$ 29. $-\frac{1}{2}$ 31. (a), (b) $\frac{2}{3}$
 35. 7 39. 6 41. -4 43. Does not exist



47. (a) (i) 2 (ii) -2 (b) No (c)



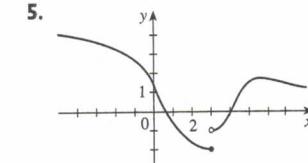
49. (a) (i) -2 (ii) Does not exist (iii) -3
 (b) (i) $n - 1$ (ii) n (c) a is not an integer.
 55. 8 61. 15; -1

EXERCISES 2.4 ■ PAGE 95

1. $\frac{4}{7}$ (or any smaller positive number)
 3. 1.44 (or any smaller positive number)
 5. 0.0906 (or any smaller positive number)
 7. $0.11, 0.012$ (or smaller positive numbers)
 9. (a) 0.031 (b) 0.010
 11. (a) $\sqrt{1000/\pi}$ cm (b) Within approximately 0.0445 cm
 (c) Radius; area; $\sqrt{1000/\pi}; 1000; 5; \approx 0.0445$
 13. (a) 0.025 (b) 0.0025
 35. (a) 0.093 (b) $\delta = (B^{2/3} - 12)/(6B^{1/3}) - 1$, where
 $B = 216 + 108\varepsilon + 12\sqrt{336 + 324\varepsilon + 81\varepsilon^2}$
 41. Within 0.1

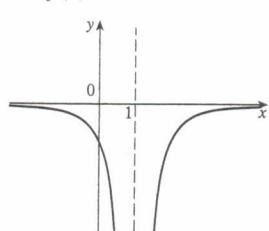
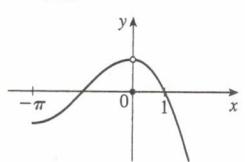
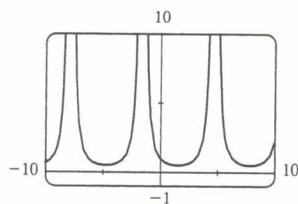
EXERCISES 2.5 ■ PAGE 105

1. $\lim_{x \rightarrow 4} f(x) = f(4)$
 3. (a) $f(-4)$ is not defined and $\lim_{x \rightarrow a} f(x)$ [for $a = -2, 2$, and 4] does not exist
 (b) -4 , neither; -2 , left; 2 , right; 4 , right

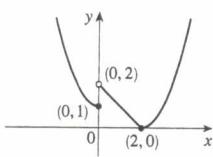


7. (a)
-
- (b) Discontinuous at $t = 1, 2, 3, 4$

9. 6

15. $f(1)$ is not defined.19. $\lim_{x \rightarrow 0} f(x) \neq f(0)$ 21. $\{x \mid x \neq -3, -2\}$ 23. $\left[\frac{1}{2}, \infty\right)$ 25. \mathbb{R} 27. $[0, \infty)$ 29. $x = (-\pi/2) + 2n\pi$, n an integer31. $\frac{7}{3}$ 33. $\pi/8$

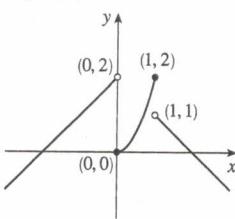
37. 0, left

41. $\frac{2}{3}$ 43. (a) $g(x) = x^3 + x^2 + x + 1$ (b) $g(x) = x^2 + x$

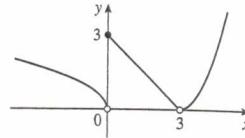
51. (b) (0.86, 0.87) 53. (b) 1.434

59. None 61. Yes

39. 0, right; 1, left



39. 0, right; 1, left

23. (a) (i) 3 (ii) 0 (iii) Does not exist (iv) 0 (v) 0 (vi) 0
(b) At 0 and 3 (c)25. $[0, \infty)$

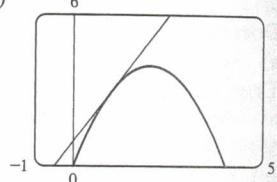
29. 0

PROBLEMS PLUS ■ PAGE 110

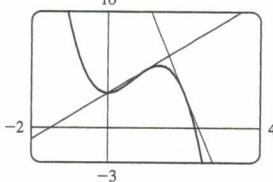
1. $\frac{2}{3}$ 3. -4 5. (a) Does not exist. (b) 17. $a = \frac{1}{2} \pm \frac{1}{2}\sqrt{5}$ 9. $\frac{3}{4}$ 11. (b) Yes (c) Yes; no

CHAPTER 3

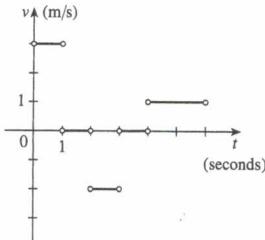
EXERCISES 3.1 ■ PAGE 119

1. (a) $\frac{f(x) - f(3)}{x - 3}$ (b) $\lim_{x \rightarrow 3} \frac{f(x) - f(3)}{x - 3}$ 3. (a) 2 (b) $y = 2x + 1$ (c)5. $y = -x + 5$ 7. $y = \frac{1}{2}x + \frac{1}{2}$ 9. (a) $8a - 6a^2$ (b) $y = 2x + 3$, $y = -8x + 19$

(c)

11. (a) Right: $0 < t < 1$ and $4 < t < 6$; left: $2 < t < 3$; standing still: $1 < t < 2$ and $3 < t < 4$

(b)

13. -24 ft/s 15. $-2/a^3$ m/s; -2 m/s; $-\frac{1}{4}$ m/s; $-\frac{2}{27}$ m/s17. $g'(0), 0, g'(4), g'(2), g'(-2)$

CHAPTER 2 REVIEW ■ PAGE 108

True-False Quiz

1. False 3. True 5. False 7. True 9. False
-
11. True 13. True 15. True

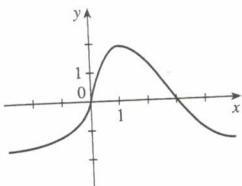
Exercises

1. (a) (i) 3 (ii) 0 (iii) Does not exist (iv) 2

(v) ∞ (vi) $-\infty$ (b) $x = 0, x = 2$ (c) $-3, 0, 2, 4$ 3. 1 5. $\frac{3}{2}$ 7. 3 9. ∞ 11. $\frac{4}{7}$ 13. $-\frac{1}{8}$

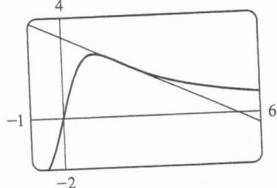
15. 0 17. 1

19.



21. 7; $y = 7x - 12$

23. (a) $-\frac{3}{5}$; $y = -\frac{3}{5}x + \frac{16}{5}$ (b)



25. $-2 + 8a$

27. $\frac{5}{(a+3)^2}$

29. $\frac{-1}{2(a+2)^{3/2}}$

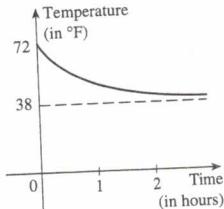
31. $f(x) = x^{10}$, $a = 1$ or $f(x) = (1+x)^{10}$, $a = 0$

33. $f(x) = 2^x$, $a = 5$

35. $f(x) = \cos x$, $a = \pi$ or $f(x) = \cos(\pi + x)$, $a = 0$

37. 1 m/s; 1 m/s

39. Greater (in magnitude)



41. (a) (i) 11 percent/year (ii) 13 percent/year

(iii) 16 percent/year

(b) 14.5 percent/year (c) 15 percent/year

43. (a) (i) \$20.25/unit (ii) \$20.05/unit (b) \$20/unit

45. (a) The rate at which the cost is changing per ounce of gold produced; dollars per ounce

(b) When the 800th ounce of gold is produced, the cost of production is \$17/oz.

(c) Decrease in the short term; increase in the long term

47. The rate at which the temperature is changing at 10:00 AM;

4°F/h

49. (a) The rate at which the oxygen solubility changes with respect to the water temperature; (mg/L)/°C

(b) $S'(16) \approx -0.25$; as the temperature increases past 16°C, the oxygen solubility is decreasing at a rate of 0.25 (mg/L)/°C.

51. Does not exist

EXERCISES 3.2 ■ PAGE 131

1. (a) 1.5

(b) 1

(c) 0

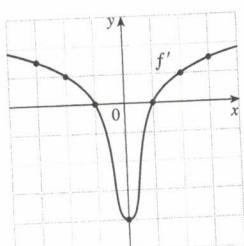
(d) -4

(e) 0

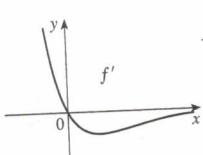
(f) 1

(g) 1.5

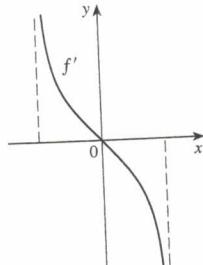
3. (a) II (b) IV (c) I (d) III



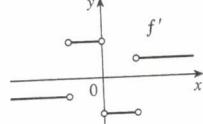
5.



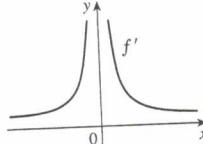
7.



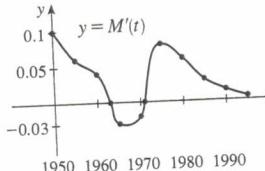
9.



11.



13.



1963 to 1971

15. (a) 0, 1, 2, 4 (b) -1, -2, -4 (c) $f'(x) = 2x$

17. $f'(x) = \frac{1}{2}$, \mathbb{R} , \mathbb{R} 19. $f'(t) = 5 - 18t$, \mathbb{R} , \mathbb{R}

21. $f'(x) = 3x^2 - 3$, \mathbb{R} , \mathbb{R}

23. $g'(x) = 1/\sqrt{1+2x}$, $[-\frac{1}{2}, \infty)$, $(-\frac{1}{2}, \infty)$

25. $G'(t) = \frac{4}{(t+1)^2}$, $(-\infty, -1) \cup (-1, \infty)$, $(-\infty, -1) \cup (-1, \infty)$

27. $f'(x) = 4x^3$, \mathbb{R} , \mathbb{R} 29. (a) $f'(x) = 4x^3 + 2$

31. (a) The rate at which the unemployment rate is changing, in percent unemployed per year

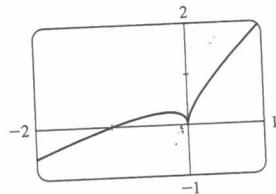
(b)

t	$U'(t)$	t	$U'(t)$
1993	-0.80	1998	-0.35
1994	-0.65	1999	-0.25
1995	-0.35	2000	0.25
1996	-0.35	2001	0.90
1997	-0.45	2002	1.10

33. -4 (corner); 0 (discontinuity)

35. -1 (vertical tangent); 4 (corner)

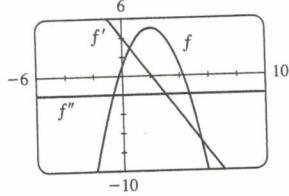
37.

Differentiable at -1;
not differentiable at 0

39. $a = f$, $b = f'$, $c = f''$

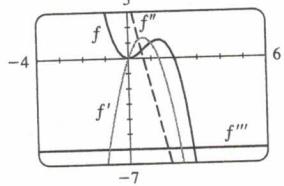
41. a = acceleration, b = velocity, c = position

43.



$$f'(x) = 4 - 2x, \\ f''(x) = -2$$

45.

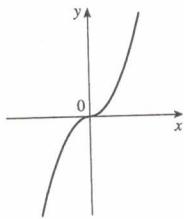


$$f'(x) = 4x - 3x^2, \\ f''(x) = 4 - 6x, \\ f'''(x) = -6, \\ f^{(4)}(x) = 0$$

$$49. f'(x) = \begin{cases} -1 & \text{if } x < 6 \\ 1 & \text{if } x > 6 \end{cases}$$

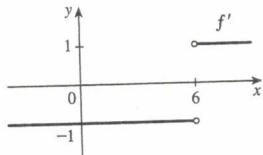
$$\text{or } f'(x) = \frac{x-6}{|x-6|}$$

51. (a)



(b) All x

$$(c) f'(x) = 2|x|$$



55. 63°

EXERCISES 3.3 ■ PAGE 144

$$1. f'(x) = 0 \quad 3. f'(t) = -\frac{2}{3} \quad 5. f'(x) = 3x^2 - 4$$

$$7. f'(t) = t^3 \quad 9. V'(r) = 4\pi r^2 \quad 11. Y'(t) = -54t^{-10}$$

$$13. F'(x) = \frac{5}{32}x^4 \quad 15. A'(s) = 60/s^6 \quad 17. y' = 0$$

$$19. u' = \frac{1}{5}t^{-4/5} + 10t^{3/2} \quad 21. y' = 5x^4 + 3x^2 + 2x$$

$$23. V'(x) = 14x^6 - 4x^3 - 6 \quad 25. F'(y) = 5 + 14/y^2 + 9/y^4$$

$$27. g'(x) = 5/(2x+1)^2 \quad 29. y' = \frac{x^2(3-x^2)}{(1-x^2)^2}$$

$$31. y' = 2v - 1/\sqrt{v} \quad 33. y' = \frac{2t(-t^4 - 4t^2 + 7)}{(t^4 - 3t^2 + 1)^2}$$

$$35. y' = 2ax + b \quad 37. y' = \frac{r(4 + 3\sqrt{r})}{2(1 + \sqrt{r})^2}$$

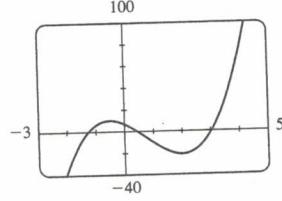
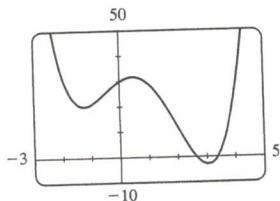
$$39. y' = (7t^3 + 4t^2 - 2)/(3t^{5/3}) \quad 41. f'(x) = 2cx/(x^2 + c)^2$$

$$43. P'(x) = na_n x^{n-1} + (n-1)a_{n-1} x^{n-2} + \dots + 2a_2 x + a_1$$

$$45. 45x^{14} - 15x^2$$

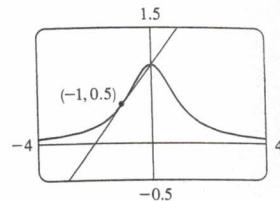
47. (a)

$$(c) 4x^3 - 9x^2 - 12x + 7$$



$$49. y = \frac{1}{2}x + \frac{1}{2}$$

$$51. (a) y = \frac{1}{2}x + 1 \quad (b)$$



$$53. y = \frac{3}{2}x + \frac{1}{2}, y = -\frac{2}{3}x + \frac{8}{3} \quad 55. y = -\frac{1}{2}x + \frac{5}{2}, y = 2x$$

$$57. f'(x) = 4x^3 - 9x^2 + 16, f''(x) = 12x^2 - 18x$$

$$59. \frac{2x^2 + 2x}{(1 + 2x)^2}, \frac{2}{(1 + 2x)^3}$$

$$61. (a) v(t) = 3t^2 - 3, a(t) = 6t \quad (b) 12 \text{ m/s}^2$$

$$(c) a(1) = 6 \text{ m/s}^2$$

$$63. (a) -16 \quad (b) -\frac{20}{9} \quad (c) 20$$

$$65. 16 \quad 67. (a) 0 \quad (b) -\frac{2}{3}$$

$$69. (a) y' = xg'(x) + g(x) \quad (b) y' = [g(x) - xg'(x)]/[g(x)]^2$$

$$(c) y' = [xg'(x) - g(x)]/[g(x)]^2$$

$$71. (-2, 21), (1, -6) \quad 75. y = 12x - 15, y = 12x + 17$$

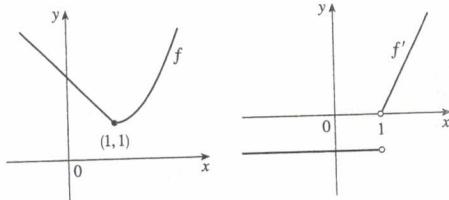
$$77. y = \frac{1}{3}x - \frac{1}{3} \quad 79. (\pm 2, 4)$$

$$81. (c) 3(x^4 + 3x^3 + 17x + 82)^2(4x^3 + 9x^2 + 17)$$

$$83. P(x) = x^2 - x + 3 \quad 85. y = \frac{3}{16}x^3 - \frac{9}{4}x + 3$$

87. \$1.627 billion/year

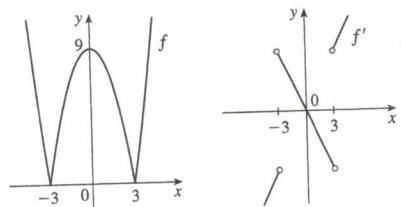
89. No



91. (a) Not differentiable at 3 or -3

$$f'(x) = \begin{cases} 2x & \text{if } |x| > 3 \\ -2x & \text{if } |x| < 3 \end{cases}$$

(b)



$$93. a = -\frac{1}{2}, b = 2$$

$$95. 6 \quad 99. 1000 \quad 101. 3, 1$$

EXERCISES 3.4 ■ PAGE 154

$$1. f'(x) = 6x + 2 \sin x \quad 3. f'(x) = \cos x - \frac{1}{2} \csc^2 x$$

$$5. g'(t) = 3t^2 \cos t - t^3 \sin t$$

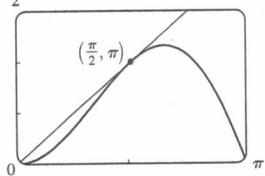
$$7. h'(\theta) = \csc \theta - \theta \csc \theta \cot \theta + \csc^2 \theta$$

$$9. y' = \frac{2 - \tan x + x \sec^2 x}{(2 - \tan^2 x)^2} \quad 11. f'(\theta) = \frac{\sec \theta \tan \theta}{(1 + \sec \theta)^2}$$

$$13. y' = (x \cos x - 2 \sin x)/x^3 \quad 15. y' = \sec \theta (\sec^2 \theta + \tan^2 \theta)$$

$$21. y = 2\sqrt{3}x - \frac{2}{3}\sqrt{3}\pi + 2 \quad 23. y = x + 1$$

25. (a) $y = 2x$ (b) $\frac{3\pi}{2}$



27. (a) $\sec x \tan x - 1$

29. $\theta \cos \theta + \sin \theta; 2 \cos \theta - \theta \sin \theta$

31. (a) $f'(x) = (1 + \tan x)/\sec x$ (b) $f'(x) = \cos x + \sin x$

33. $(2n+1)\pi \pm \frac{1}{3}\pi$, n an integer

35. (a) $v(t) = 8 \cos t$, $a(t) = -8 \sin t$

(b) $4\sqrt{3}, -4, -4\sqrt{3}$; to the left

37. 5 ft/rad 39. 3 41. 3 43. $\sin 1$

45. $\frac{1}{2}$ 47. $-\sqrt{2}$

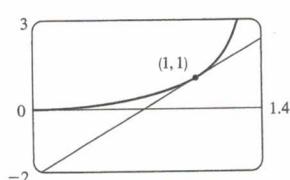
49. (a) $\sec^2 x = 1/\cos^2 x$ (b) $\sec x \tan x = (\sin x)/\cos^2 x$

(c) $\cos x - \sin x = (\cot x - 1)/\csc x$

51. 1

EXERCISES 3.5 ■ PAGE 161

1. $4 \cos 4x$
3. $-20x(1-x^2)^9$
5. $\frac{\cos x}{2\sqrt{\sin x}}$
7. $F'(x) = 10x(x^4 + 3x^2 - 2)^4(2x^2 + 3)$
9. $F'(x) = \frac{2 + 3x^2}{4(1 + 2x + x^3)^{3/4}}$
11. $g'(t) = -\frac{12t^3}{(t^4 + 1)^4}$
13. $y' = -3x^2 \sin(x^3 + x^3)$
15. $y' = \sec kx(kx \tan kx + 1)$
17. $g'(x) = 4(1 + 4x)^4(3 + x - x^2)^7(17 + 9x - 21x^2)$
19. $y' = 8(2x-5)^3(8x^2-5)^{-4}(-4x^2+30x-5)$
21. $y' = \frac{-12x(x^2+1)^2}{(x^2-1)^4}$
23. $y' = (\cos x - x \sin x)\cos(x \cos x)$
25. $F'(z) = 1/[(z-1)^{1/2}(z+1)^{3/2}]$
27. $y' = (r^2+1)^{-3/2}$ 29. $y' = 2 \cos(\tan 2x) \sec^2(2x)$
31. $y' = (x \cos \sqrt{1+x^2})/\sqrt{1+x^2}$ 33. $y' = 4 \sec^2 x \tan x$
35. $y' = \frac{16 \sin 2x(1 - \cos 2x)^3}{(1 + \cos 2x)^5}$
37. $y' = -2 \cos \theta \cot(\sin \theta) \csc^2(\sin \theta)$
39. $y' = 3[x^2 + (1 - 3x)^5]^2[2x - 15(1 - 3x)^4]$
41. $y' = \frac{1 + 1/(2\sqrt{x})}{2\sqrt{x} + \sqrt{x}}$
43. $g'(x) = p(2r \sin rx + n)^{p-1}(2r^2 \cos rx)$
45. $y' = \frac{-\pi \cos(\tan \pi x) \sec^2(\pi x) \sin \sqrt{\sin(\tan \pi x)}}{2\sqrt{\sin(\tan \pi x)}}$
47. $h'(x) = x/\sqrt{x^2 + 1}$, $h''(x) = 1/(x^2 + 1)^{3/2}$
49. $H'(t) = 3 \sec^2 3t$, $H''(t) = 18 \sec^2 3t \tan 3t$
51. $y = 20x + 1$ 53. $y = -x + \pi$
55. (a) $y = \pi x - \pi + 1$ (b)



57. (a) $f'(x) = (2 - 2x^2)/\sqrt{2 - x^2}$

59. $((\pi/2) + 2n\pi, 3)$, $((3\pi/2) + 2n\pi, -1)$, n an integer

61. 24

63. (a) 30 (b) 36

65. (a) $\frac{3}{4}$ (b) Does not exist (c) -2

67. (a) $F'(x) = -\sin x f'(\cos x)$ (b) $G'(x) = -\sin(f(x))f'(x)$

69. 120 71. 96 73. $2^{103} \sin 2x$

75. $v(t) = \frac{5}{2}\pi \cos(10\pi t)$ cm/s

77. (a) $\frac{dB}{dt} = \frac{7\pi}{54} \cos \frac{2\pi t}{5.4}$ (b) 0.16

79. dv/dt is the rate of change of velocity with respect to time;
 dv/ds is the rate of change of velocity with respect to displacement

81. (b) The factored form

85. (b) $-n \cos^{n-1} x \sin[(n+1)x]$

EXERCISES 3.6 ■ PAGE 169

1. (a) $y' = -(y+2+6x)/x$

(b) $y = (4/x) - 2 - 3x$, $y' = -(4/x^2) - 3$

3. (a) $y' = -y^2/x^2$ (b) $y = x/(x-1)$, $y' = -1/(x-1)^2$

5. $y' = -x^2/y^2$

7. $y' = \frac{2x+y}{2y-x}$ 9. $y' = \frac{3y^2 - 5x^4 - 4x^3y}{x^4 + 3y^2 - 6xy}$

11. $y' = \frac{-2xy^2 - \sin y}{2x^2y + x \cos y}$ 13. $y' = \tan x \tan y$

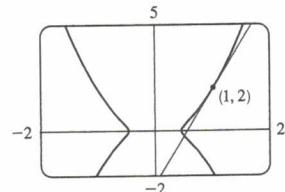
15. $y' = \frac{y \sec^2(x/y) - y^2}{y^2 + x \sec^2(x/y)}$ 17. $y' = \frac{4xy\sqrt{xy} - y}{x - 2x^2\sqrt{xy}}$

19. $y' = \frac{y \sin x + y \cos(xy)}{\cos x - x \cos(xy)}$ 21. $-\frac{16}{13}$

23. $x' = \frac{-2x^4y + x^3 - 6xy^2}{4x^3y^2 - 3x^2y + 2y^3}$ 25. $y = -x + 2$

27. $y = x + \frac{1}{2}$ 29. $y = -\frac{9}{13}x + \frac{40}{13}$

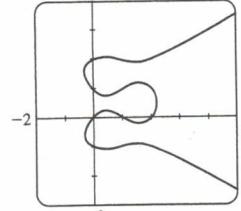
31. (a) $y = \frac{9}{2}x - \frac{5}{2}$ (b)



33. $-81/y^3$

35. $-2x/y^5$

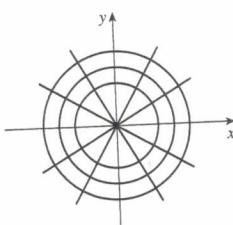
37. (a) Eight; $x \approx 0.42, 1.58$



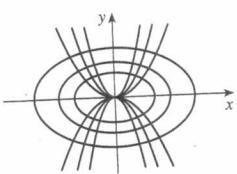
(b) $y = -x + 1$, $y = \frac{1}{3}x + 2$ (c) $1 \mp \frac{1}{3}\sqrt{3}$

39. $(\pm \frac{5}{4}\sqrt{3}, \pm \frac{5}{4})$ 41. $(x_0x/a^2) - (y_0y/b^2) = 1$

45.



47.



49. $(\pm\sqrt{3}, 0)$

51. $(-1, -1), (1, 1)$

53. 2

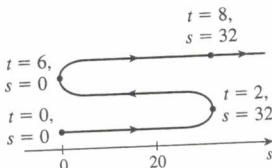
EXERCISES 3.7 ■ PAGE 179

1. (a) $3t^2 - 24t + 36$ (b) -9 ft/s (c) $t = 2, 6$

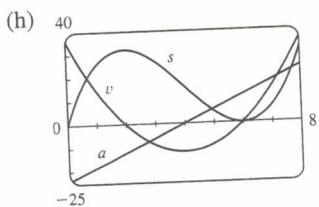
(d) $0 \leq t < 2, t > 6$

(e) 96 ft

(f)



(g) $6t - 24; -6 \text{ m/s}^2$

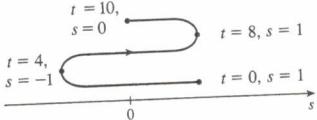


(i) Speeding up when $2 < t < 4$ or $t > 6$; slowing down when $0 \leq t < 2$ or $4 < t < 6$

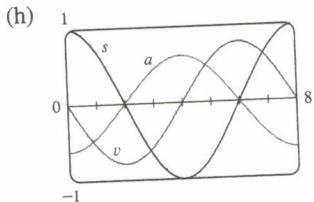
3. (a) $-\frac{\pi}{4} \sin\left(\frac{\pi t}{4}\right)$ (b) $-\frac{1}{8}\pi\sqrt{2} \text{ ft/s}$ (c) $t = 0, 4, 8$

(d) $4 < t < 8$ (e) 4 ft

(f)



(g) $-\frac{1}{16}\pi^2 \cos(\pi t/4); \frac{1}{32}\pi^2\sqrt{2} \text{ ft/s}^2$



(i) Speeding up when $0 < t < 2, 4 < t < 6, 8 < t < 10$; slowing down when $2 < t < 4, 6 < t < 8$

5. (a) Speeding up when $0 < t < 1$ or $2 < t < 3$; slowing down when $1 < t < 2$

(b) Speeding up when $1 < t < 2$ or $3 < t < 4$; slowing down when $0 < t < 1$ or $2 < t < 3$

7. (a) $t = 4 \text{ s}$

(b) $t = 1.5 \text{ s}$; the velocity has an absolute minimum.

9. (a) 5.02 m/s (b) $\sqrt{17} \text{ m/s}$

11. (a) $30 \text{ mm}^2/\text{mm}$; the rate at which the area is increasing with respect to side length as x reaches 15 mm

(b) $\Delta A \approx 2x \Delta x$

13. (a) (i) 5π (ii) 4.5π (iii) 4.1π

(b) 4π (c) $\Delta A \approx 2\pi r \Delta r$

15. (a) $8\pi \text{ ft}^2/\text{ft}$ (b) $16\pi \text{ ft}^2/\text{ft}$ (c) $24\pi \text{ ft}^2/\text{ft}$
The rate increases as the radius increases.

17. (a) 6 kg/m (b) 12 kg/m (c) 18 kg/m
At the right end; at the left end

19. (a) 4.75 A (b) $5 \text{ A}; t = \frac{2}{3} \text{ s}$

21. (a) $dV/dP = -C/P^2$ (b) At the beginning

23. (a) 16 million/year; 78.5 million/year
(b) $P(t) = at^3 + bt^2 + ct + d$, where $a \approx 0.00129371$,
 $b \approx -7.061422$, $c \approx 12,822.979$, $d \approx -7,743,770$
(c) $P'(t) = 3at^2 + 2bt + c$
(d) 14.48 million/year; 75.29 million/year (smaller)
(e) 81.62 million/year

25. (a) $0.926 \text{ cm/s}; 0.694 \text{ cm/s}; 0$

(b) $0; -92.6 \text{ (cm/s)/cm}; -185.2 \text{ (cm/s)/cm}$

(c) At the center; at the edge

27. (a) $C'(x) = 12 - 0.2x + 0.0015x^2$

(b) \$32/yard; the cost of producing the 201st yard

(c) \$32.20

29. (a) $[xp'(x) - p(x)]/x^2$; the average productivity increases as new workers are added.

31. -0.2436 K/min

33. (a) 0 and 0 (b) $C = 0$

(c) $(0, 0), (500, 50)$; it is possible for the species to coexist.

EXERCISES 3.8 ■ PAGE 186

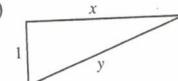
1. $dV/dt = 3x^2 dx/dt$ 3. $48 \text{ cm}^2/\text{s}$ 5. $3/(25\pi) \text{ m/min}$

7. 70 9. $\pm\frac{46}{13}$

11. (a) The plane's altitude is 1 mi and its speed is 500 mi/h.

(b) The rate at which the distance from the plane to the station is increasing when the plane is 2 mi from the station

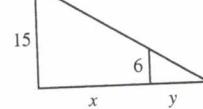
(c) $y^2 = x^2 + 1$ (d) $y^2 = x^2 + 1$
(e) $250\sqrt{3} \text{ mi/h}$



13. (a) The height of the pole (15 ft), the height of the man (6 ft), and the speed of the man (5 ft/s)

(b) The rate at which the tip of the man's shadow is moving when he is 40 ft from the pole

(c) $\frac{15}{6} = \frac{x+y}{y}$ (d) $\frac{15}{6} = \frac{x+y}{y}$ (e) $\frac{25}{3} \text{ ft/s}$



15. 65 mi/h 17. $837/\sqrt{8674} \approx 8.99 \text{ ft/s}$

19. -1.6 cm/min 21. $\frac{720}{13} \approx 55.4 \text{ km/h}$

23. $(10,000 + 800,000\pi/9) \approx 2.89 \times 10^5 \text{ cm}^3/\text{min}$

25. $\frac{10}{3} \text{ cm/min}$ 27. $6/(5\pi) \approx 0.38 \text{ ft/min}$ 29. $0.3 \text{ m}^2/\text{s}$

31. $80 \text{ cm}^3/\text{min}$ 33. $\frac{107}{810} \approx 0.132 \Omega/\text{s}$ 35. 0.396 m/min

37. (a) 360 ft/s (b) 0.096 rad/s 39. $\frac{10}{9}\pi \text{ km/min}$

41. $1650/\sqrt{31} \approx 296 \text{ km/h}$ 43. $\frac{7}{4}\sqrt{15} \approx 6.78 \text{ m/s}$

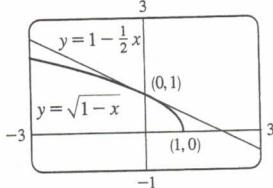
EXERCISES 3.9 • PAGE 193

1. $L(x) = -10x - 6$ 3. $L(x) = -x + \pi/2$

5. $\sqrt{1-x} \approx 1 - \frac{1}{2}x$;

$\sqrt{0.9} \approx 0.95$,

$\sqrt{0.99} \approx 0.995$



7. $-1.204 < x < 0.706$ 9. $-0.045 < x < 0.055$

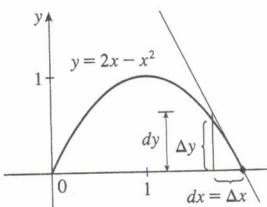
11. (a) $dy = 2x(x \cos 2x + \sin 2x) dx$ (b) $dy = \frac{t}{\sqrt{1+t^2}} dt$

13. (a) $dy = \frac{-2}{(u-1)^2} du$ (b) $dy = -\frac{6r^2}{(1+r^3)^3} dr$

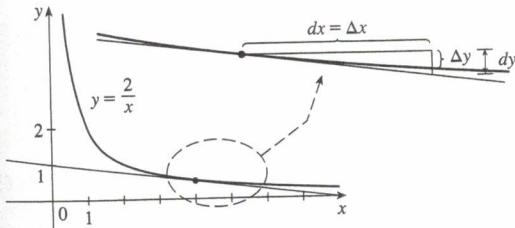
15. (a) $dy = 5/(2\sqrt{4+5x}) dx$ (b) 0.05

17. (a) $dy = \sec^2 x dx$ (b) -0.2

19. $\Delta y = 0.64$, $dy = 0.8$



21. $\Delta y = -0.1$, $dy = -0.125$



23. 32.08 25. 4.02 27. $1 - \pi/90 \approx 0.965$

31. (a) 270 cm^3 , 0.01, 1% (b) 36 cm^2 , 0.006, 0.6%

33. (a) $84/\pi \approx 27 \text{ cm}^2$; $\frac{1}{84} \approx 0.012$

(b) $1764/\pi^2 \approx 179 \text{ cm}^3$; $\frac{1}{56} \approx 0.018$

35. (a) $2\pi rh \Delta r$ (b) $\pi(\Delta r)^2 h$

41. (a) 4.8, 5.2 (b) Too large

CHAPTER 3 REVIEW • PAGE 196

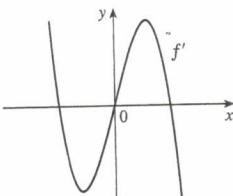
True-False Quiz

1. False 3. False 5. True 7. False 9. True
11. False

Exercises

1. (a) (i) 3 m/s (ii) 2.75 m/s (iii) 2.625 m/s
(iv) 2.525 m/s (b) 2.5 m/s

3.



5. $a = f$, $c = f'$, $b = f''$

7. (a) The rate at which the cost changes with respect to the interest rate; dollars/(percent per year)
(b) As the interest rate increases past 10%, the cost is increasing at a rate of \$1200/(percent per year).
(c) Always positive

9. The rate at which the total value of US currency in circulation is changing in billions of dollars per year; \$22.2 billion/year

11. $f'(x) = 3x^2 + 5$ 13. $6x(x^4 - 3x^2 + 5)^2(2x^2 - 3)$

15. $\frac{1}{2\sqrt{x}} - \frac{4}{3\sqrt[3]{x^7}}$ 17. $\frac{2(2x^2 + 1)}{\sqrt{x^2 + 1}}$ 19. $\frac{t^2 + 1}{(1-t^2)^2}$

21. $-(\sec^2 \sqrt{1-x})/(2\sqrt{1-x})$ 23. $\frac{1-y^4-2xy}{4xy^3+x^2-3}$

25. $\frac{2 \sec 2\theta (\tan 2\theta - 1)}{(1 + \tan 2\theta)^2}$ 27. $-(x-1)^{-2}$

29. $\frac{2x - y \cos(xy)}{x \cos(xy) + 1}$ 31. $-6x \csc^2(3x^2 + 5)$

33. $\frac{\cos \sqrt{x} - \sqrt{x} \sin \sqrt{x}}{2\sqrt{x}}$ 35. $2 \cos \theta \tan(\sin \theta) \sec^2(\sin \theta)$

37. $\frac{1}{5}(x \tan x)^{-4/5}(\tan x + x \sec^2 x)$

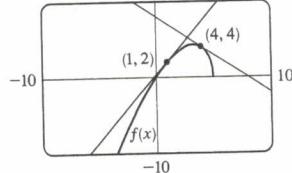
39. $\cos(\tan \sqrt{1+x^3})(\sec^2 \sqrt{1+x^3}) \frac{3x^2}{2\sqrt{1+x^3}}$ 41. $-\frac{4}{27}$

43. $-5x^4/y^{11}$ 45. 1 47. $y = 2\sqrt{3}x + 1 - \pi\sqrt{3}/3$

49. $y = 2x + 1$

51. (a) $\frac{10-3x}{2\sqrt{5-x}}$ (b) $y = \frac{7}{4}x + \frac{1}{4}$, $y = -x + 8$

(c)



53. $(\pi/4, \sqrt{2}), (5\pi/4, -\sqrt{2})$ 55. $y = -\frac{2}{3}x^2 + \frac{14}{3}x$

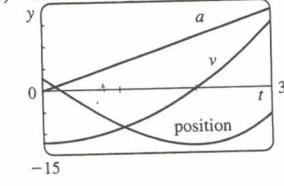
59. (a) 2 (b) 44 61. $2xg(x) + x^2g'(x)$ 63. $2g(x)g'(x)$

65. $f'(x) = g'(g(x))g'(x)$ 67. $f'(x) = g'(\sin x) \cdot \cos x$

69. $\frac{f'(x)[g(x)]^2 + g'(x)[f(x)]^2}{[f(x) + g(x)]^2}$

71. $f'(g(\sin 4x))g'(\sin 4x)(\cos 4x)(4)$

73. (a) $v(t) = 3t^2 - 12$; $a(t) = 6t$ (b) $t > 2$; $0 \leq t < 2$
(c) 23 (d) 20 (e) $t > 2$; $0 < t < 2$



75. 4 kg/m 77. $\frac{4}{3} \text{ cm}^2/\text{min}$ 79. 13 ft/s 81. 400 ft/h

83. (a) $L(x) = 1 + x$; $\sqrt[3]{1+3x} \approx 1+x$; $\sqrt[3]{1.03} \approx 1.01$
 (b) $-0.23 < x < 0.40$
 85. $12 + \frac{3}{2}\pi \approx 16.7 \text{ cm}^2$ 87. $\frac{1}{32}$ 89. $\frac{1}{4}$ 91. $\frac{1}{8}x^2$

PROBLEMS PLUS ■ PAGE 200

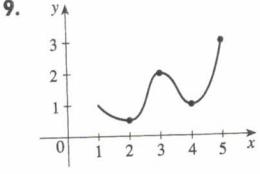
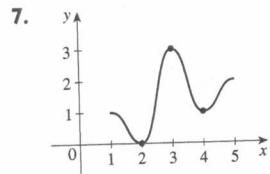
1. $(\pm \frac{1}{2}\sqrt{3}, \frac{1}{4})$
 5. (a) 0 (b) 1 (c) $f'(x) = x^2 + 1$
 9. $(0, \frac{5}{4})$
 11. (a) $4\pi\sqrt{3}/\sqrt{11} \text{ rad/s}$ (b) $40(\cos\theta + \sqrt{8 + \cos^2\theta}) \text{ cm}$
 (c) $-480\pi \sin\theta(1 + \cos\theta/\sqrt{8 + \cos^2\theta}) \text{ cm/s}$
 13. $x_T \in (3, \infty)$, $y_T \in (2, \infty)$, $x_N \in (0, \frac{5}{3})$, $y_N \in (-\frac{5}{2}, 0)$
 15. (b) (i) 53° (or 127°) (ii) 63° (or 117°)
 17. R approaches the midpoint of the radius AO .
 19. $-\sin a$ 21. $(1, -2)$, $(-1, 0)$
 23. $\sqrt{29}/58$ 25. $2 + \frac{375}{128}\pi \approx 11.204 \text{ cm}^3/\text{min}$

CHAPTER 4

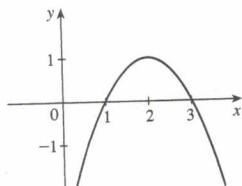
EXERCISES 4.1 ■ PAGE 211

Abbreviations: abs., absolute; loc., local; max., maximum; min., minimum

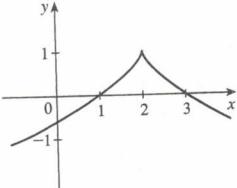
1. Absolute minimum: smallest function value on the entire domain of the function; local minimum at c : smallest function value when x is near c
 3. Abs. max. at s , abs. min. at r , loc. max. at c , loc. min. at b and r
 5. Abs. max. $f(4) = 5$, loc. max. $f(4) = 5$ and $f(6) = 4$, loc. min. $f(2) = 2$ and $f(1) = f(5) = 3$



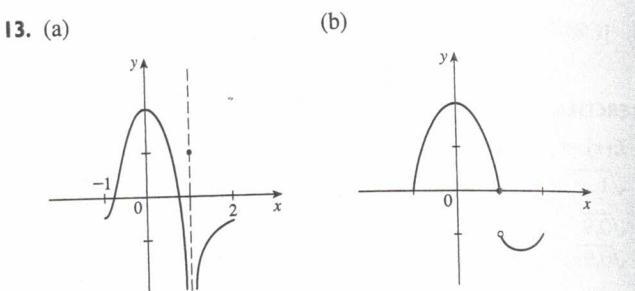
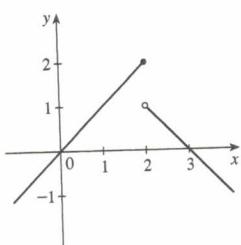
11. (a)



(b)



(c)



15. Abs. max. $f(1) = 5$ 17. None
 19. Abs. min. $f(0) = 0$
 21. Abs. max. $f(-3) = 9$, abs. and loc. min. $f(0) = 0$
 23. None 25. Abs. max. $f(0) = 1$
 27. Abs. max. $f(3) = 2$ 29. $-\frac{2}{5}$ 31. $-4, 2$
 33. $0, \frac{1}{2}(-1 \pm \sqrt{5})$ 35. $0, 2$ 37. $0, \frac{4}{9}$ 39. $0, \frac{8}{7}, 4$
 41. $n\pi$ (n an integer) 43. 10 45. $f(0) = 5, f(2) = -7$
 47. $f(-1) = 8, f(2) = -19$
 49. $f(3) = 66, f(\pm 1) = 2$ 51. $f(1) = \frac{1}{2}, f(0) = 0$
 53. $f(\sqrt{2}) = 2, f(-1) = -\sqrt{3}$
 55. $f(\pi/6) = \frac{3}{2}\sqrt{3}, f(\pi/2) = 0$

57. $f\left(\frac{a}{a+b}\right) = \frac{a^ab^b}{(a+b)^{a+b}}$

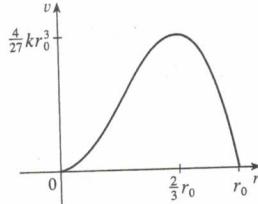
59. (a) 2.19, 1.81 (b) $\frac{6}{25}\sqrt{\frac{3}{5}} + 2, -\frac{6}{25}\sqrt{\frac{3}{5}} + 2$

61. (a) 0.32, 0.00 (b) $\frac{3}{16}\sqrt{3}, 0$ 63. $\approx 3.9665^\circ\text{C}$

65. Cheapest, $t \approx 0.855$ (June 1994); most expensive, $t \approx 4.618$ (March 1998)

67. (a) $r = \frac{2}{3}r_0$ (b) $v = \frac{4}{27}kr_0^3$

(c)

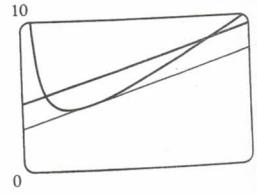
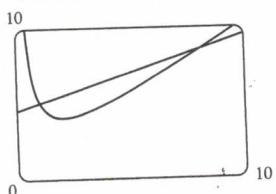


EXERCISES 4.2 ■ PAGE 219

1. 2 3. $\frac{9}{4}$ 5. f is not differentiable on $(-1, 1)$
 7. 0.8, 3.2, 4.4, 6.1

9. (a), (b)

(c) $2\sqrt{2}$



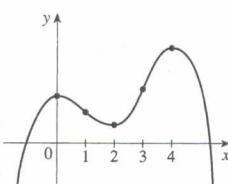
11. 0 13. $\sqrt{3}/9$ 15. f is not continuous at 3
 23. 16 25. No 31. No

EXERCISES 4.3 ■ PAGE 227

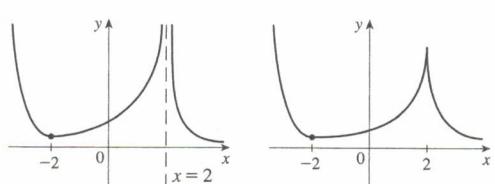
Abbreviations: inc., increasing; dec., decreasing; CD, concave downward; CU, concave upward; HA, horizontal asymptote; VA, vertical asymptote; IP, inflection point(s)

1. (a) $(1, 3), (4, 6)$ (b) $(0, 1), (3, 4)$ (c) $(0, 2)$
 (d) $(2, 4), (4, 6)$ (e) $(2, 3)$
3. (a) I/D Test (b) Concavity Test
 (c) Find points at which the concavity changes.
5. (a) Inc. on $(1, 5)$; dec. on $(0, 1)$ and $(5, 6)$
 (b) Loc. max. at $x = 5$, loc. min. at $x = 1$
7. $x = 1, 7$
9. (a) Inc. on $(-\infty, -3), (2, \infty)$; dec. on $(-3, 2)$
 (b) Loc. max. $f(-3) = 81$; loc. min. $f(2) = -44$
 (c) CU on $(-\frac{1}{2}, \infty)$; CD on $(-\infty, -\frac{1}{2})$; IP $(-\frac{1}{2}, \frac{37}{2})$
11. (a) Inc. on $(-1, 0), (1, \infty)$; dec. on $(-\infty, -1), (0, 1)$
 (b) Loc. max. $f(0) = 3$; loc. min. $f(\pm 1) = 2$
 (c) CU on $(-\infty, -\sqrt{3}/3), (\sqrt{3}/3, \infty)$;
 CD on $(-\sqrt{3}/3, \sqrt{3}/3)$; IP $(\pm \sqrt{3}/3, \frac{22}{9})$
13. (a) Inc. on $(0, \pi/4), (5\pi/4, 2\pi)$; dec. on $(\pi/4, 5\pi/4)$
 (b) Loc. max. $f(\pi/4) = \sqrt{2}$; loc. min. $f(5\pi/4) = -\sqrt{2}$
 (c) CU on $(3\pi/4, 7\pi/4)$; CD on $(0, 3\pi/4), (7\pi/4, 2\pi)$;
 IP $(3\pi/4, 0), (7\pi/4, 0)$
15. Loc. max. $f(-1) = 7$, loc. min. $f(1) = -1$
17. Loc. max. $f(\frac{3}{4}) = \frac{5}{4}$
19. (a) f has a local maximum at 2.
 (b) f has a horizontal tangent at 6.

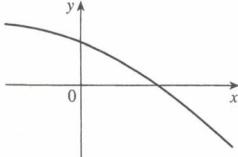
21.



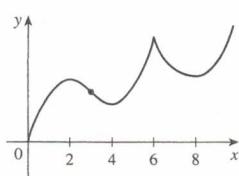
23.



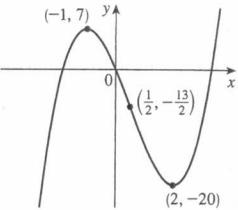
25.



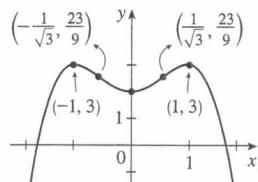
27. (a) Inc. on $(0, 2), (4, 6), (8, \infty)$;
 dec. on $(2, 4), (6, 8)$
 (b) Loc. max. at $x = 2, 6$;
 loc. min. at $x = 4, 8$
 (c) CU on $(3, 6), (6, \infty)$;
 CD on $(0, 3)$
 (d) 3 (e) See graph at right.



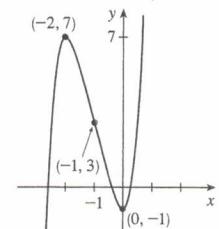
29. (a) Inc. on $(-\infty, -1), (2, \infty)$;
 dec. on $(-1, 2)$
 (b) Loc. max. $f(-1) = 7$;
 loc. min. $f(2) = -20$
 (c) CU on $(\frac{1}{2}, \infty)$; CD on $(-\infty, \frac{1}{2})$;
 IP $(\frac{1}{2}, -\frac{13}{2})$
 (d) See graph at right.



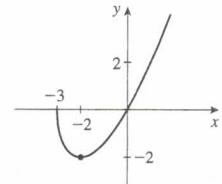
31. (a) Inc. on $(-\infty, -1), (0, 1)$;
 dec. on $(-1, 0), (1, \infty)$
 (b) Loc. max. $f(-1) = 3, f(1) = 3$;
 loc. min. $f(0) = 2$
 (c) CU on $(-1/\sqrt{3}, 1/\sqrt{3})$;
 CD on $(-\infty, -1/\sqrt{3}), (1/\sqrt{3}, \infty)$;
 IP $(\pm 1/\sqrt{3}, \frac{23}{9})$
 (d) See graph at right.



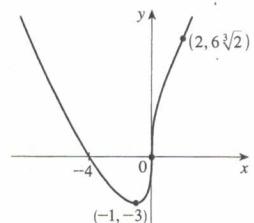
33. (a) Inc. on $(-\infty, -2), (0, \infty)$;
 dec. on $(-2, 0)$
 (b) Loc. max. $h(-2) = 7$;
 loc. min. $h(0) = -1$
 (c) CU on $(-1, \infty)$;
 CD on $(-\infty, -1)$; IP $(-1, 3)$
 (d) See graph at right.



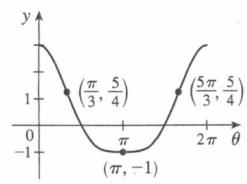
35. (a) Inc. on $(-2, \infty)$;
 dec. on $(-3, -2)$
 (b) Loc. min. $A(-2) = -2$
 (c) CU on $(-3, \infty)$
 (d) See graph at right.



37. (a) Inc. on $(-1, \infty)$;
 dec. on $(-\infty, -1)$
 (b) Loc. min. $C(-1) = -3$
 (c) CU on $(-\infty, 0), (2, \infty)$;
 CD on $(0, 2)$;
 IPs $(0, 0), (2, 6\sqrt[3]{2})$
 (d) See graph at right.



39. (a) Inc. on $(\pi, 2\pi)$;
 dec. on $(0, \pi)$
 (b) Loc. min. $f(\pi) = -1$
 (c) CU on $(\pi/3, 5\pi/3)$;
 CD on $(0, \pi/3), (5\pi/3, 2\pi)$;
 IP $(\pi/3, \frac{5}{4}), (5\pi/3, \frac{5}{4})$
 (d) See graph at right.

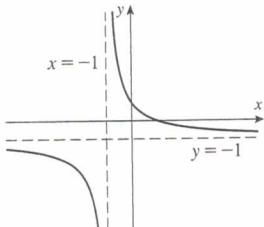


41. $(3, \infty)$
43. (a) Loc. and abs. max. $f(1) = \sqrt{2}$, no min.
 (b) $\frac{1}{4}(3 - \sqrt{17})$
45. (b) CU on $(0.94, 2.57), (3.71, 5.35)$;
 CD on $(0, 0.94), (2.57, 3.71), (5.35, 2\pi)$;
 IP $(0.94, 0.44), (2.57, -0.63), (3.71, -0.63), (5.35, 0.44)$
47. CU on $(-\infty, -0.6), (0, 0, \infty)$; CD on $(-0.6, 0, 0)$

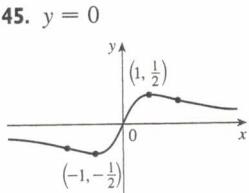
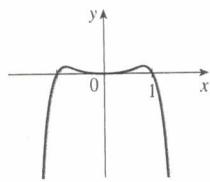
49. (a) The rate of increase is initially very small, increases to a maximum at $t \approx 8$ h, then decreases toward 0.
 (b) When $t = 8$ (c) CU on $(0, 8)$; CD on $(8, 18)$ (d) $(8, 350)$
 51. $K(3) - K(2)$; CD 53. $f(x) = \frac{1}{9}(2x^3 + 3x^2 - 12x + 7)$

EXERCISES 4.4 ■ PAGE 240

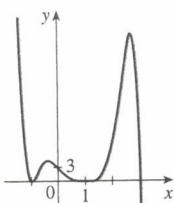
1. (a) As x becomes large, $f(x)$ approaches 5.
 (b) As x becomes large negative, $f(x)$ approaches 3.
 3. (a) ∞ (b) ∞ (c) $-\infty$ (d) 1 (e) 2
 (f) $x = -1$, $x = 2$, $y = 1$, $y = 2$
 5. 0 7. $\frac{3}{2}$ 9. 0 11. $-\frac{1}{2}$ 13. $\frac{1}{2}$ 15. 2
 17. 3 19. $\frac{1}{6}$ 21. $\frac{1}{2}(a - b)$ 23. ∞ 25. $-\infty$
 27. ∞ 29. 1
 31. (a), (b) $-\frac{1}{2}$ 33. $y = 2$; $x = 2$
 35. $y = 2$; $x = -2$, $x = 1$ 37. $x = 5$ 39. $y = 3$
 41. $f(x) = \frac{2-x}{x^2(x-3)}$
 43. $y = -1$



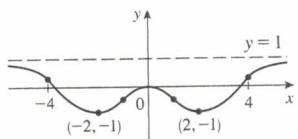
47. $-\infty, -\infty$



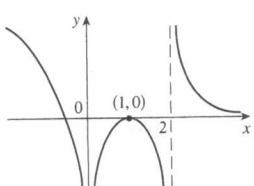
49. $-\infty, \infty$



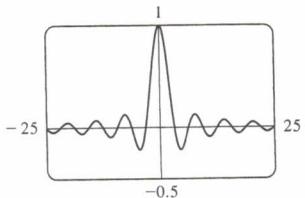
51.



53.



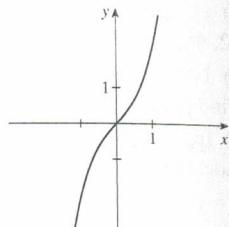
55. (a) 0 (b) An infinite number of times



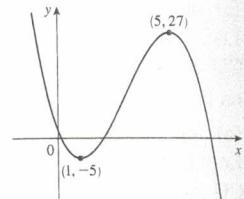
57. (a) 0 (b) $\pm\infty$ 59. 4
 61. $N \geq 15$ 63. $N \leq -6, N \leq -22$ 65. (a) $x > 100$

EXERCISES 4.5 ■ PAGE 248

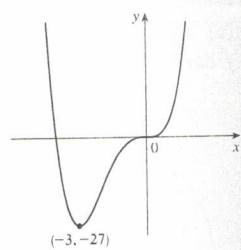
1. A. \mathbb{R} B. y-int. 0; x-int. 0
 C. About $(0, 0)$ D. None
 E. Inc. on $(-\infty, \infty)$ F. None
 G. CU on $(0, \infty)$; CD on $(-\infty, 0)$; IP $(0, 0)$
 H. See graph at right.



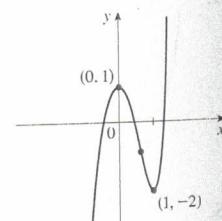
3. A. \mathbb{R} B. y-int. 2; x-int. $2, \frac{1}{2}(7 \pm 3\sqrt{5})$
 C. None D. None
 E. Inc. on $(1, 5)$; dec. on $(-\infty, 1), (5, \infty)$
 F. Loc. min. $f(1) = -5$; loc. max. $f(5) = 27$
 G. CU on $(-\infty, 3)$; CD on $(3, \infty)$; IP $(3, 11)$
 H. See graph at right.



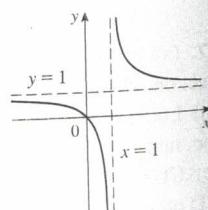
5. A. \mathbb{R} B. y-int. 0; x-int. $-4, 0$
 C. None D. None
 E. Inc. on $(-3, \infty)$; dec. on $(-\infty, -3)$
 F. Loc. min. $f(-3) = -27$
 G. CU on $(-\infty, -2), (0, \infty)$; CD on $(-2, 0)$; IP $(0, 0), (-2, -16)$
 H. See graph at right.



7. A. \mathbb{R} B. y-int. 1
 C. None D. None
 E. Inc. on $(-\infty, 0), (1, \infty)$; dec. on $(0, 1)$
 F. Loc. max. $f(0) = 1$; loc. min. $f(1) = -2$
 G. CU on $(1/\sqrt[3]{4}, \infty)$; CD on $(-\infty, 1/\sqrt[3]{4})$; IP $(1/\sqrt[3]{4}, 1 - 9/(2\sqrt[3]{16}))$
 H. See graph at right.

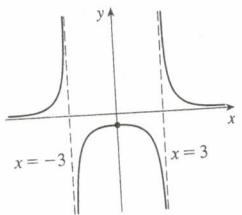


9. A. $\{x \mid x \neq 1\}$ B. y-int. 0; x-int. 0
 C. None D. VA $x = 1$, HA $y = 1$
 E. Dec. on $(-\infty, 1), (1, \infty)$
 F. None
 G. CU on $(1, \infty)$; CD on $(-\infty, 1)$
 H. See graph at right.

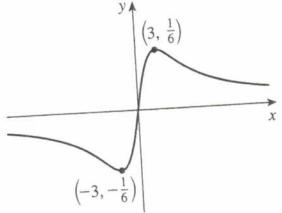


APPENDIX H ANSWERS TO ODD-NUMBERED EXERCISES

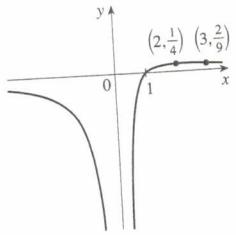
11. A. $\{x \mid x \neq \pm 3\}$ B. y-int. $-\frac{1}{9}$
 C. About y-axis D. VA $x = \pm 3$, HA $y = 0$
 E. Inc. on $(-\infty, -3)$, $(-3, 0)$; dec. on $(0, 3)$, $(3, \infty)$
 F. Loc. max. $f(0) = -\frac{1}{9}$
 G. CU on $(-\infty, -3)$, $(3, \infty)$; CD on $(-3, 3)$
 H. See graph at right.



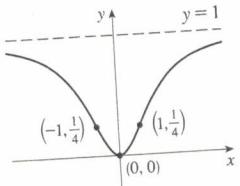
13. A. \mathbb{R} B. y-int. 0; x-int. 0
 C. About $(0, 0)$ D. HA $y = 0$
 E. Inc. on $(-3, 3)$; dec. on $(-\infty, -3)$, $(3, \infty)$
 F. Loc. min. $f(-3) = -\frac{1}{6}$; loc. max. $f(3) = \frac{1}{6}$
 G. CU on $(-3\sqrt{3}, 0)$, $(3\sqrt{3}, \infty)$; CD on $(-\infty, -3\sqrt{3})$, $(0, 3\sqrt{3})$; IP $(0, 0)$, $(\pm 3\sqrt{3}, \pm\sqrt{3}/12)$
 H. See graph at right.



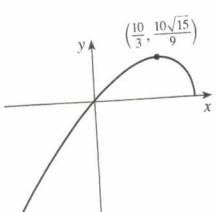
15. A. $(-\infty, 0) \cup (0, \infty)$ B. x-int. 1
 C. None D. HA $y = 0$; VA $x = 0$
 E. Inc. on $(0, 2)$; dec. on $(-\infty, 0)$, $(2, \infty)$
 F. Loc. max. $f(2) = \frac{1}{4}$
 G. CU on $(3, \infty)$; CD on $(-\infty, 0)$, $(0, 3)$; IP $(3, \frac{2}{9})$
 H. See graph at right.



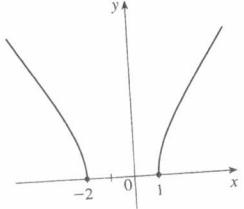
17. A. \mathbb{R} B. y-int. 0, x-int. 0
 C. About y-axis D. HA $y = 1$
 E. Inc. on $(0, \infty)$; dec. on $(-\infty, 0)$
 F. Loc. min. $f(0) = 0$
 G. CU on $(-1, 1)$; CD on $(-\infty, -1)$, $(1, \infty)$; IP $(\pm 1, \frac{1}{4})$
 H. See graph at right.



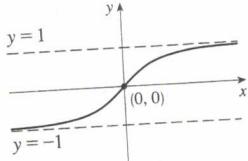
19. A. $(-\infty, 5]$ B. y-int. 0; x-int. 0, 5
 C. None D. None
 E. Inc. on $(-\infty, \frac{10}{3})$; dec. on $(\frac{10}{3}, 5)$
 F. Loc. max. $f(\frac{10}{3}) = \frac{10}{9}\sqrt{15}$
 G. CD on $(-\infty, 5)$
 H. See graph at right.



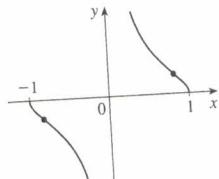
21. A. $(-\infty, -2) \cup (1, \infty)$
 B. x-int. -2, 1
 C. None D. None
 E. Inc. on $(1, \infty)$; dec. on $(-\infty, -2)$
 F. None
 G. CD on $(-\infty, -2)$, $(1, \infty)$
 H. See graph at right.



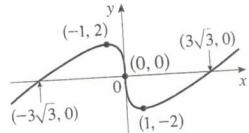
23. A. \mathbb{R} B. y-int. 0; x-int. 0
 C. About the origin
 D. HA $y = \pm 1$
 E. Inc. on $(-\infty, \infty)$ F. None
 G. CU on $(-\infty, 0)$; CD on $(0, \infty)$; IP $(0, 0)$
 H. See graph at right.



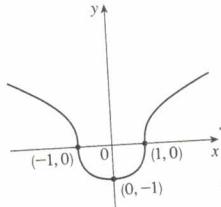
25. A. $\{x \mid |x| \leq 1, x \neq 0\} = [-1, 0) \cup (0, 1]$
 B. x-int. ± 1 C. About $(0, 0)$
 D. VA $x = 0$
 E. Dec. on $(-1, 0)$, $(0, 1)$
 F. None
 G. CU on $(-1, -\sqrt{2}/3)$, $(0, \sqrt{2}/3)$; CD on $(-\sqrt{2}/3, 0)$, $(\sqrt{2}/3, 1)$; IP $(\pm\sqrt{2}/3, \pm 1/\sqrt{2})$
 H. See graph at right.



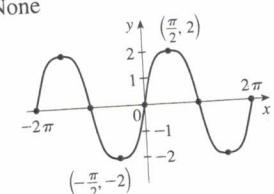
27. A. \mathbb{R} B. y-int. 0; x-int. 0, $\pm 3\sqrt{3}$ C. About the origin
 D. None E. Inc. on $(-\infty, -1)$, $(1, \infty)$; dec. on $(-1, 1)$
 F. Loc. max. $f(-1) = 2$; loc. min. $f(1) = -2$
 G. CU on $(0, \infty)$; CD on $(-\infty, 0)$; IP $(0, 0)$
 H. See graph at right.



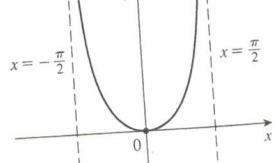
29. A. \mathbb{R} B. y-int. -1; x-int. ± 1
 C. About y-axis D. None
 E. Inc. on $(0, \infty)$; dec. on $(-\infty, 0)$
 F. Loc. min. $f(0) = -1$
 G. CU on $(-1, 1)$; CD on $(-\infty, -1)$, $(1, \infty)$; IP $(\pm 1, 0)$
 H. See graph at right.



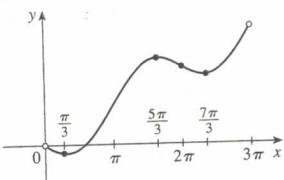
31. A. \mathbb{R} B. y-int. 0; x-int. $n\pi$ (n an integer)
 C. About the origin, period 2π D. None
 E. Inc. on $(2n\pi - \pi/2, 2n\pi + \pi/2)$; dec. on $(2n\pi + \pi/2, 2n\pi + 3\pi/2)$
 F. Loc. max. $f(2n\pi + \pi/2) = 2$; loc. min. $f(2n\pi + 3\pi/2) = -2$
 G. CU on $((2n-1)\pi, 2n\pi)$; CD on $(2n\pi, (2n+1)\pi)$; IP $(n\pi, 0)$
 H. See graph at right.



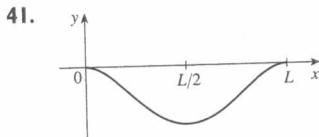
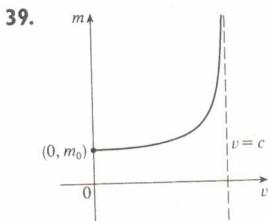
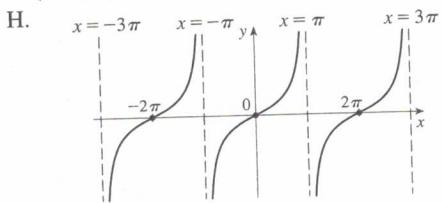
33. A. $(-\pi/2, \pi/2)$ B. y-int. 0; x-int. 0 C. About y-axis
 D. VA $x = \pm\pi/2$
 E. Inc. on $(0, \pi/2)$; dec. on $(-\pi/2, 0)$
 F. Loc. min. $f(0) = 0$
 G. CU on $(-\pi/2, \pi/2)$
 H. See graph at right.



35. A. $(0, 3\pi)$ C. None D. None
 E. Inc. on $(\pi/3, 5\pi/3)$, $(7\pi/3, 3\pi)$; dec. on $(0, \pi/3)$, $(5\pi/3, 7\pi/3)$
 F. Loc. min. $f(\pi/3) = (\pi/6) - \frac{1}{2}\sqrt{3}$, $f(7\pi/3) = (7\pi/6) - \frac{1}{2}\sqrt{3}$; loc. max. $f(5\pi/3) = (5\pi/6) + \frac{1}{2}\sqrt{3}$
 G. CU on $(0, \pi)$, $(2\pi, 3\pi)$; CD on $(\pi, 2\pi)$; IP $(\pi, \pi/2)$, $(2\pi, \pi)$
 H. See graph at right.

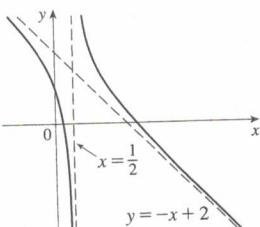


37. A. All reals except $(2n+1)\pi$ (n an integer)
 B. y-int. 0; x-int. $2n\pi$
 C. About the origin, period 2π
 D. VA $x = (2n+1)\pi$
 E. Inc. on $((2n-1)\pi, (2n+1)\pi)$ F. None
 G. CU on $(2n\pi, (2n+1)\pi)$; CD on $((2n-1)\pi, 2n\pi)$
 H. IP $(2n\pi, 0)$

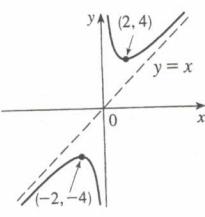


43. $y = x - 1$ 45. $y = 2x - 2$

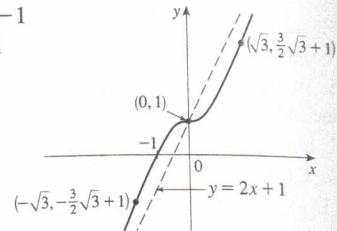
47. A. $(-\infty, \frac{1}{2}) \cup (\frac{1}{2}, \infty)$
 B. y-int. 1; x-int. $\frac{1}{4}(5 \pm \sqrt{17})$
 C. None
 D. VA $x = \frac{1}{2}$; SA $y = -x + 2$
 E. Dec. on $(-\infty, \frac{1}{2})$, $(\frac{1}{2}, \infty)$
 F. None
 G. CU on $(\frac{1}{2}, \infty)$; CD on $(-\infty, \frac{1}{2})$
 H. See graph at right



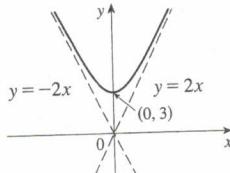
49. A. $\{x | x \neq 0\}$ B. None
 C. About $(0, 0)$ D. VA $x = 0$; SA $y = x$
 E. Inc. on $(-\infty, -2)$, $(2, \infty)$; dec. on $(-2, 0)$, $(0, 2)$
 F. Loc. max. $f(-2) = -4$; loc. min. $f(2) = 4$
 G. CU on $(0, \infty)$; CD on $(-\infty, 0)$
 H. See graph at right.



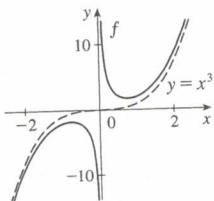
51. A. \mathbb{R} B. y-int. 1; x-int. -1
 C. None D. SA $y = 2x + 1$
 E. Inc. on $(-\infty, \infty)$ F. None
 G. CU on $(-\infty, -\sqrt{3})$, $(0, \sqrt{3})$; CD on $(-\sqrt{3}, 0)$, $(\sqrt{3}, \infty)$; IP $(\pm\sqrt{3}, 1 \pm \frac{3}{2}\sqrt{3})$, $(0, 1)$
 H. See graph at right.



53.

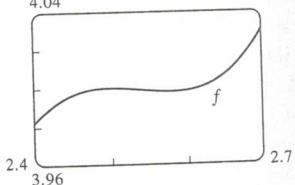
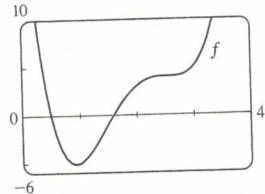


57. VA $x = 0$, asymptotic to $y = x^3$

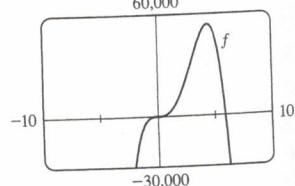
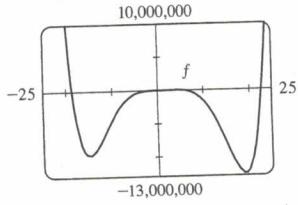


EXERCISES 4.6 ■ PAGE 255

1. Inc. on $(0.92, 2.5)$, $(2.58, \infty)$; dec. on $(-\infty, 0.92)$, $(2.5, 2.58)$; loc. max. $f(2.5) \approx 4$; loc. min. $f(0.92) \approx -5.12$, $f(2.58) \approx 3.998$; CU on $(-\infty, 1.46)$, $(2.54, \infty)$; CD on $(1.46, 2.54)$; IP $(1.46, -1.40)$, $(2.54, 3.999)$

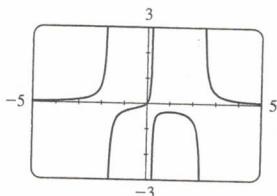


3. Inc. on $(-15, 4.40)$, $(18.93, \infty)$; dec. on $(-\infty, -15)$, $(4.40, 18.93)$; loc. max. $f(4.40) \approx 53,800$; loc. min. $f(-15) \approx -9,700,000$, $f(18.93) \approx -12,700,000$; CU on $(-\infty, -11.34)$, $(0, 2.92)$, $(15.08, \infty)$; CD on $(-11.34, 0)$, $(2.92, 15.08)$; IP $(0, 0)$, $\approx (-11.34, -6,250,000)$, $(2.92, 31,800)$, $(15.08, -8,150,000)$

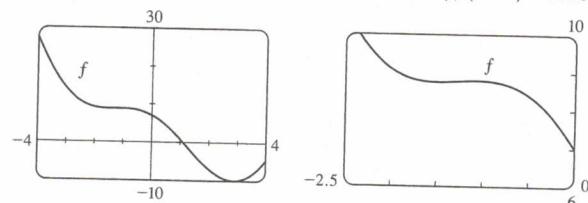


5. Inc. on $(-\infty, -1.7)$, $(-1.7, 0.24)$, $(0.24, 1)$; dec. on $(1, 2.46)$, $(2.46, \infty)$; loc. max. $f(1) = -\frac{1}{3}$

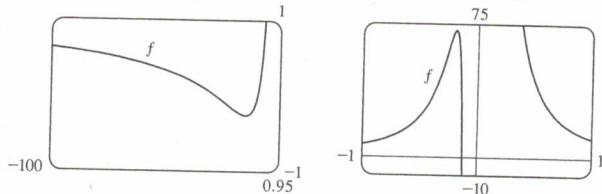
CU on $(-\infty, -1.7), (-0.506, 0.24), (2.46, \infty)$;
 CD on $(-1.7, -0.506), (0.24, 2.46)$; IP $(-0.506, -0.192)$



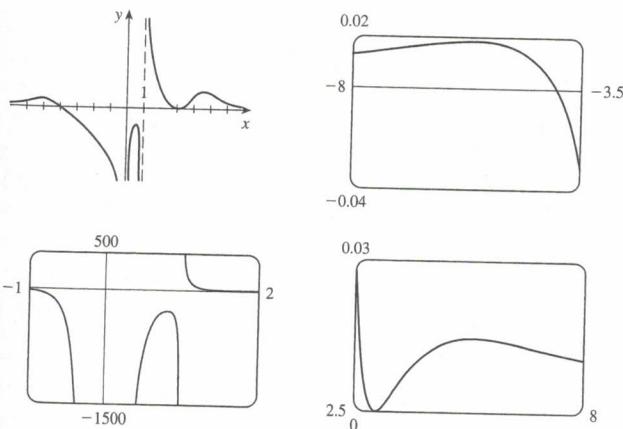
7. Inc. on $(-1.49, -1.07), (2.89, 4)$; dec. on $(-4, -1.49), (-1.07, 2.89)$; loc. max. $f(-1.07) \approx 8.79$; loc. min. $f(-1.49) \approx 8.75, f(2.89) \approx -9.99$; CU on $(-4, -1.28), (1.28, 4)$; CD on $(-1.28, 1.28)$; IP $(-1.28, 8.77), (1.28, -1.48)$



9. Inc. on $(-8 - \sqrt{61}, -8 + \sqrt{61})$; dec. on $(-\infty, -8 - \sqrt{61}), (-8 + \sqrt{61}, 0), (0, \infty)$; CU on $(-12 - \sqrt{138}, -12 + \sqrt{138}), (0, \infty)$; CD on $(-\infty, -12 - \sqrt{138}), (-12 + \sqrt{138}, 0)$



11. Loc. max. $f(-5.6) \approx 0.018, f(0.82) \approx -281.5, f(5.2) \approx 0.0145$; loc. min. $f(3) = 0$

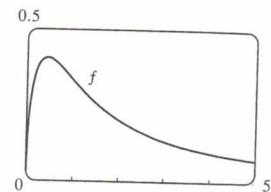


$$13. f'(x) = -\frac{x(x+1)^2(x^3+18x^2-44x-16)}{(x-2)^3(x-4)^5}$$

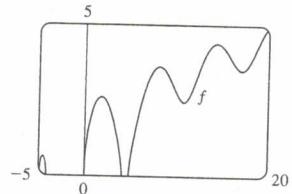
$$f''(x) = 2\frac{(x+1)(x^6+36x^5+6x^4-628x^3+684x^2+672x+64)}{(x-2)^4(x-4)^6}$$

CU on $(-35.3, -5.0), (-1, -0.5), (-0.1, 2), (2, 4), (4, \infty)$;
 CD on $(-\infty, -35.3), (-5.0, -1), (-0.5, -0.1)$;
 IP $(-35.3, -0.015), (-5.0, -0.005), (-1, 0), (-0.5, 0.00001), (-0.1, 0.0000066)$

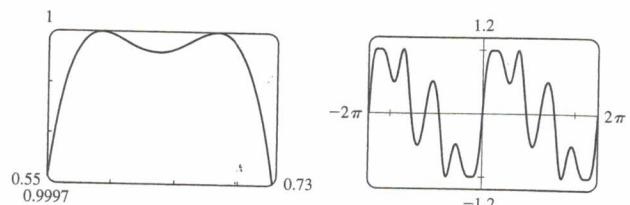
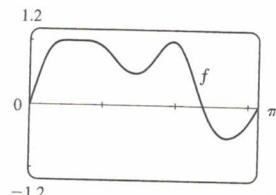
15. Inc. on $(0, 0.43)$; dec. on $(0.43, \infty)$; loc. max. $f(0.43) \approx 0.41$; CU on $(0.94, \infty)$; CD on $(0, 0.94)$; IP $(0.94, 0.34)$



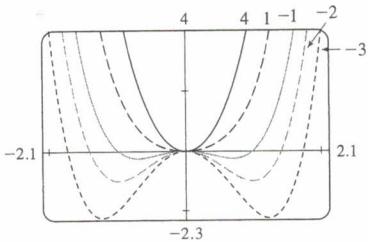
17. Inc. on $(-4.91, -4.51), (0, 1.77), (4.91, 8.06), (10.79, 14.34), (17.08, 20)$;
 dec. on $(-4.51, -4.10), (1.77, 4.10), (8.06, 10.79), (14.34, 17.08)$;
 loc. max. $f(-4.51) \approx 0.62, f(1.77) \approx 2.58, f(8.06) \approx 3.60, f(14.34) \approx 4.39$;
 loc. min. $f(10.79) \approx 2.43, f(17.08) \approx 3.49$; CU on $(9.60, 12.25), (15.81, 18.65)$;
 CD on $(-4.91, -4.10), (0, 4.10), (4.91, 9.60), (12.25, 15.81), (18.65, 20)$;
 IPs at $(9.60, 2.95), (12.25, 3.27), (15.81, 3.91), (18.65, 4.20)$



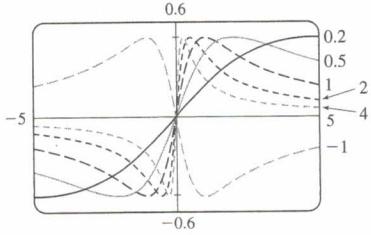
19. Max. $f(0.59) \approx 1, f(0.68) \approx 1, f(1.96) \approx 1$;
 min. $f(0.64) \approx 0.99996, f(1.46) \approx 0.49, f(2.73) \approx -0.51$;
 IP $(0.61, 0.99998), (0.66, 0.99998), (1.17, 0.72), (1.75, 0.77), (2.28, 0.34)$



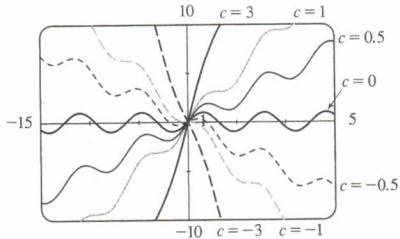
- 21.** For $c \geq 0$, there is no IP and only one extreme point, the origin. For $c < 0$, there is a maximum point at the origin, two minimum points, and two IPs, which move downward and away from the origin as $c \rightarrow -\infty$.



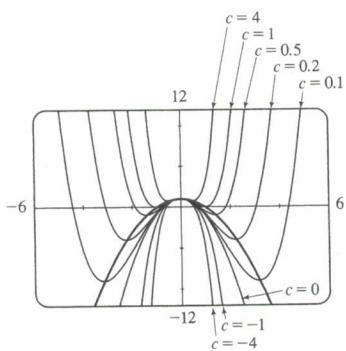
- 23.** For $c > 0$, the maximum and minimum values are always $\pm \frac{1}{2}$, but the extreme points and IPs move closer to the y-axis as c increases. $c = 0$ is a transitional value: when c is replaced by $-c$, the curve is reflected in the x-axis.



- 25.** For $|c| < 1$, the graph has local maximum and minimum values; for $|c| \geq 1$ it does not. The function increases for $c \geq 1$ and decreases for $c \leq -1$. As c changes, the IPs move vertically but not horizontally.

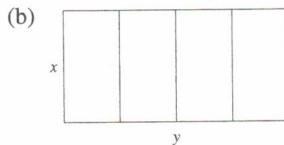
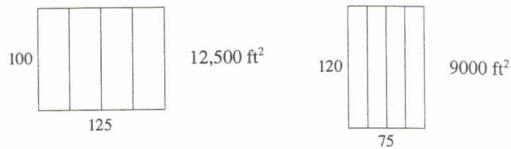
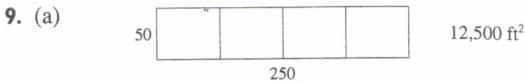


- 27.** (a) Positive (b)



EXERCISES 4.7 ■ PAGE 262

1. (a) 11, 12 (b) 11.5, 11.5 3. 10, 10
5. 25 m by 25 m 7. $N = 1$



- (c) $A = xy$ (d) $5x + 2y = 750$ (e) $A(x) = 375x - \frac{5}{2}x^2$
(f) 14,062.5 ft²

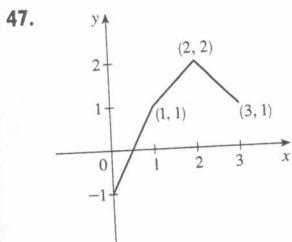
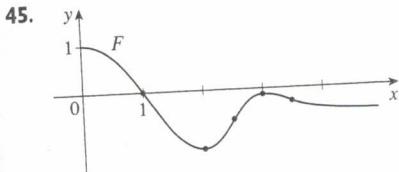
11. 1000 ft by 1500 ft 13. 4000 cm³ 15. \$191.28
17. $(-\frac{28}{17}, \frac{7}{17})$ 19. $(-\frac{1}{3}, \pm \frac{4}{3}\sqrt{2})$ 21. Square, side $\sqrt{2}r$
23. $L/2, \sqrt{3}L/4$ 25. Base $\sqrt{3}r$, height $3r/2$
27. $4\pi r^3/(3\sqrt{3})$ 29. $\pi r^2(1 + \sqrt{5})$ 31. 24 cm, 36 cm
33. (a) Use all of the wire for the square
(b) $40\sqrt{3}/(9 + 4\sqrt{3})$ m for the square
35. Height = radius = $\sqrt[3]{V/\pi}$ cm 37. $V = 2\pi R^3/(9\sqrt{3})$
41. $E^2/(4r)$ 43. (a) $\frac{3}{2}s^2 \csc \theta (\csc \theta - \sqrt{3} \cot \theta)$ (b) $\cos^{-1}(1/\sqrt{3}) \approx 55^\circ$
(c) $6s[h + s/(2\sqrt{2})]$
45. Row directly to B 47. ≈ 4.85 km east of the refinery
49. $10\sqrt[3]{3}/(1 + \sqrt[3]{3})$ ft from the stronger source
51. $(a^{2/3} + b^{2/3})^{3/2}$
53. (b) (i) \$342,491; \$342/unit; \$390/unit (ii) 400
(iii) \$320/unit
55. (a) $p(x) = 19 - \frac{1}{3000}x$ (b) \$9.50
57. (a) $p(x) = 550 - \frac{1}{10}x$ (b) \$175 (c) \$100
61. 9.35 m 65. $x = 6$ in. 67. $\pi/6$ 69. $\frac{1}{2}(L + W)^2$
71. (a) About 5.1 km from B (b) C is close to B; C is close to D; $W/L = \sqrt{25 + x^2}/x$, where $x = |BC|$ (c) ≈ 1.07 ; no such value (d) $\sqrt{41}/4 \approx 1.6$

EXERCISES 4.8 ■ PAGE 272

1. (a) $x_2 \approx 2.3, x_3 \approx 3$ (b) No 3. $\frac{4}{5}$ 5. 1.1797
7. 1.1785 9. -1.25 11. 1.82056420 13. 1.217562
15. 0.876726 17. -0.724492, 1.220744
19. -0.471074, 1.461070 21. 0.641714
23. -1.93822883, -1.21997997, 1.13929375, 2.98984102
25. -1.97806681, -0.82646233
27. (b) 31.622777
33. (a) -1.293227, -0.441731, 0.507854 (b) -2.0212
35. (0.164419, 0.990958)
37. (0.410245, 0.347810)
39. 0.76286%

EXERCISES 4.9 • PAGE 279

1. $F(x) = \frac{1}{2}x^2 - 3x + C$ 3. $F(x) = \frac{1}{2}x + \frac{1}{4}x^3 - \frac{1}{5}x^4 + C$
 5. $F(x) = \frac{2}{3}x^3 + \frac{1}{2}x^2 - x + C$ 7. $F(x) = 4x^{5/4} - 4x^{7/4} + C$
 9. $F(x) = 4x^{3/2} - \frac{6}{7}x^{7/6} + C$
 11. $F(x) = \begin{cases} -5/(4x^8) + C_1 & \text{if } x < 0 \\ -5/(4x^8) + C_2 & \text{if } x > 0 \end{cases}$
 13. $F(u) = \frac{1}{3}u^3 - 6u^{-1/2} + C$
 15. $G(\theta) = \sin \theta + 5 \cos \theta + C$
 17. $F(t) = 2 \sec t + t^{1/2} + C_n$
 19. $F(x) = x^5 - \frac{1}{3}x^6 + 4$ 21. $x^3 + x^4 + Cx + D$
 23. $\frac{3}{20}x^{8/3} + Cx + D$ 25. $t^5 + \frac{1}{2}Ct^2 + Dt + E$
 27. $x - 3x^2 + 8$ 29. $4x^{3/2} + 2x^{5/2} + 4$
 31. $2 \sin t + \tan t + 4 - 2\sqrt{3}$
 33. $2x^4 + \frac{1}{3}x^3 + 5x^2 - 22x + \frac{59}{3}$
 35. $-\sin \theta - \cos \theta + 5\theta + 4$ 37. $x^2 - 2x^3 + 9x + 9$
 39. $x^2 - \cos x - \frac{1}{2}\pi x$ 41. 10 43. b



51. $s(t) = 1 - \cos t - \sin t$ 53. $s(t) = \frac{1}{6}t^3 - t^2 + 3t + 1$
 55. $s(t) = -10 \sin t - 3 \cos t + (6/\pi)t + 3$
 57. (a) $s(t) = 450 - 4.9t^2$ (b) $\sqrt{450/4.9} \approx 9.58$ s
 (c) $-9.8\sqrt{450/4.9} \approx -93.9$ m/s (d) About 9.09 s
 61. 225 ft 63. \$742.08 65. $\frac{130}{11} \approx 11.8$ s
 67. $\frac{88}{15} \approx 5.87$ ft/s² 69. $62,500 \text{ km/h}^2 \approx 4.82 \text{ m/s}^2$
 71. (a) 22.9125 mi (b) 21.675 mi (c) 30 min 33 s
 (d) 55.425 mi

CHAPTER 4 REVIEW • PAGE 282

True-False Quiz

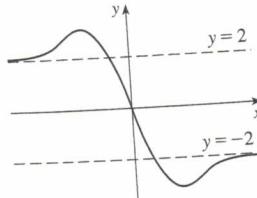
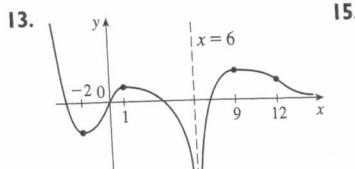
1. False 3. False 5. True 7. False 9. True
 11. True 13. False 15. True 17. True 19. True

Exercises

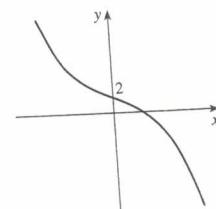
1. Abs. max. $f(4) = 5$, abs. and loc. min. $f(3) = 1$;
 loc. min. $f(3) = 1$
 3. Abs. max. $f(2) = \frac{2}{5}$, abs. and loc. min. $f(-\frac{1}{3}) = -\frac{9}{2}$

5. Abs. max. $f(\pi) = \pi$; abs. min. $f(0) = 0$; loc. max.
 $f(\pi/3) = (\pi/3) + \frac{1}{2}\sqrt{3}$; loc. min. $f(2\pi/3) = (2\pi/3) - \frac{1}{2}\sqrt{3}$

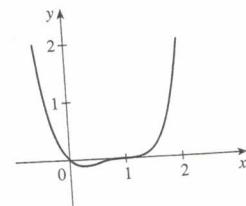
7. $\frac{1}{2}$ 9. $-\frac{2}{3}$ 11. $\frac{3}{4}$



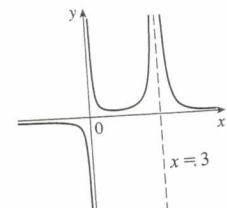
17. A. \mathbb{R} B. y-int. 2
 C. None D. None
 E. Dec. on $(-\infty, \infty)$ F. None
 G. CU on $(-\infty, 0)$;
 CD on $(0, \infty)$; IP $(0, 2)$
 H. See graph at right.



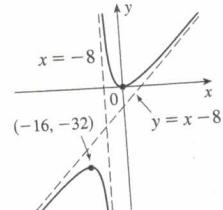
19. A. \mathbb{R} B. y-int. 0; x-int. 0, 1
 C. None D. None
 E. Inc. on $(\frac{1}{4}, \infty)$, dec. on $(-\infty, \frac{1}{4})$
 F. Loc. min. $f(\frac{1}{4}) = -\frac{27}{256}$
 G. CU on $(-\infty, \frac{1}{2})$, $(1, \infty)$;
 CD on $(\frac{1}{2}, 1)$; IP $(\frac{1}{2}, -\frac{1}{16})$, $(1, 0)$
 H. See graph at right.



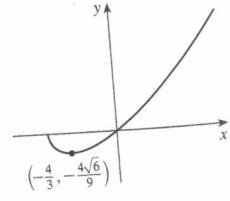
21. A. $\{x | x \neq 0, 3\}$
 B. None C. None
 D. HA $y = 0$; VA $x = 0$, $x = 3$
 E. Inc. on $(1, 3)$; dec. on $(-\infty, 0)$,
 $(0, 1)$, $(3, \infty)$
 F. Loc. min. $f(1) = \frac{1}{4}$
 G. CU on $(0, 3)$, $(3, \infty)$; CD on $(-\infty, 0)$
 H. See graph at right.



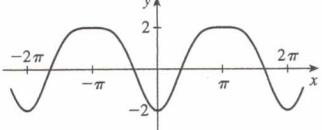
23. A. $\{x | x \neq -8\}$
 B. y-int. 0, x-int. 0 C. None
 D. VA $x = -8$; SA $y = x - 8$
 E. Inc. on $(-\infty, -16)$, $(0, \infty)$;
 dec. on $(-16, -8)$, $(-8, 0)$
 F. Loc. max. $f(-16) = -32$;
 loc. min. $f(0) = 0$
 G. CU on $(-8, \infty)$; CD on $(-\infty, -8)$
 H. See graph at right.



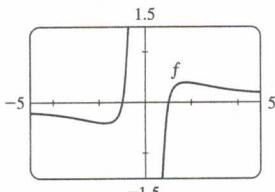
25. A. $[-2, \infty)$
 B. y-int. 0; x-int. -2, 0
 C. None D. None
 E. Inc. on $(-\frac{4}{3}, \infty)$, dec. on $(-2, -\frac{4}{3})$
 F. Loc. min. $f(-\frac{4}{3}) = -\frac{4}{9}\sqrt{6}$
 G. CU on $(-2, \infty)$
 H. See graph at right.



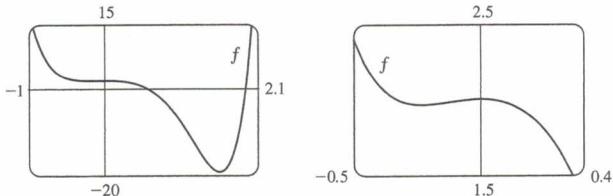
- 27.** A. \mathbb{R} B. y-int. -2
 C. About y-axis, period 2π D. None
 E. Inc. on $(2n\pi, (2n+1)\pi)$, n an integer; dec. on $((2n-1)\pi, 2n\pi)$
 F. Loc. max. $f((2n+1)\pi) = 2$; loc. min. $f(2n\pi) = -2$
 G. CU on $(2n\pi - (\pi/3), 2n\pi + (\pi/3))$;
 CD on $(2n\pi + (\pi/3), 2n\pi + (5\pi/3))$; IP $(2n\pi \pm (\pi/3), -\frac{1}{4})$
 H.



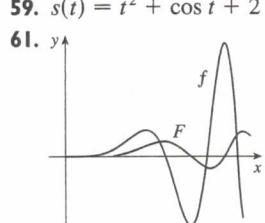
- 29.** Inc. on $(-\sqrt{3}, 0), (0, \sqrt{3})$;
 dec. on $(-\infty, -\sqrt{3}), (\sqrt{3}, \infty)$;
 loc. max. $f(\sqrt{3}) = \frac{2}{9}\sqrt{3}$,
 loc. min. $f(-\sqrt{3}) = -\frac{2}{9}\sqrt{3}$;
 CU on $(-\sqrt{6}, 0), (\sqrt{6}, \infty)$;
 CD on $(-\infty, -\sqrt{6}), (0, \sqrt{6})$;
 IP $(\sqrt{6}, \frac{5}{36}\sqrt{6}), (-\sqrt{6}, -\frac{5}{36}\sqrt{6})$



- 31.** Inc. on $(-0.23, 0), (1.62, \infty)$; dec. on $(-\infty, -0.23), (0, 1.62)$;
 loc. max. $f(0) = 2$; loc. min. $f(-0.23) \approx 1.96, f(1.62) \approx -19.2$;
 CU on $(-\infty, -0.12), (1.24, \infty)$;
 CD on $(-0.12, 1.24)$; IP $(-0.12, 1.98), (1.24, -12.1)$



- 37.** (a) 0 (b) CU on \mathbb{R} **41.** $3\sqrt{3}r^2$
43. $4/\sqrt{3}$ cm from D **45.** $L = C$ **47.** \$11.50
49. 1.297383 **51.** 1.16718557
53. $f(x) = \frac{2}{5}x^{5/2} + \frac{3}{5}x^{5/3} + C$
55. $f(t) = t^2 + 3 \cos t + 2$
57. $f(x) = \frac{1}{2}x^2 - x^3 + 4x^4 + 2x + 1$
59. $s(t) = t^2 + \cos t + 2$



- 63.** No
65. (b) About 8.5 in. by 2 in. (c) $20/\sqrt{3}$ in., $20\sqrt{2/3}$ in.

PROBLEMS PLUS ■ PAGE 286

- 5.** $(-2, 4), (2, -4)$ **7.** $\frac{4}{3}$ **9.** $(m/2, m^2/4)$
11. $-3.5 < a < -2.5$ **13.** (a) $x/(x^2 + 1)$ (b) $\frac{1}{2}$

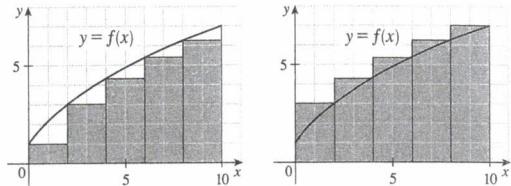
15. (a) $-\tan \theta \left[\frac{1}{c} \frac{dc}{dt} + \frac{1}{b} \frac{db}{dt} \right]$
 $b \frac{db}{dt} + c \frac{dc}{dt} - \left(b \frac{dc}{dt} + c \frac{db}{dt} \right) \sec \theta$
 (b) $\frac{\sqrt{b^2 + c^2 - 2bc \cos \theta}}{\sqrt{b^2 + c^2}}$

- 17.** (a) $T_1 = D/c_1, T_2 = (2h \sec \theta)/c_1 + (D - 2h \tan \theta)/c_2$,
 $T_3 = \sqrt{4h^2 + D^2}/c_1$
 (c) $c_1 \approx 3.85$ km/s, $c_2 \approx 7.66$ km/s, $h \approx 0.42$ km
21. $3(\sqrt[3]{2} - 1) \approx 11\frac{1}{2}$ h

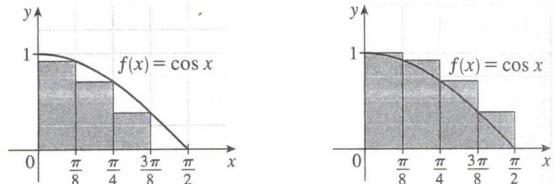
CHAPTER 5

EXERCISES 5.1 ■ PAGE 298

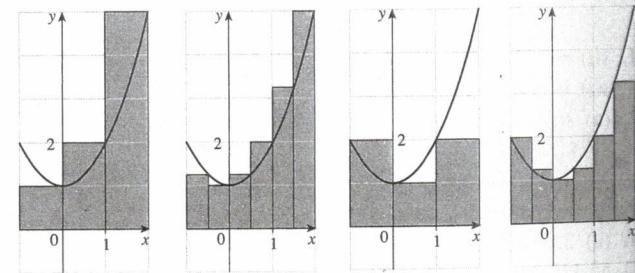
- 1.** (a) 40, 52 (b) 43.2, 49.2



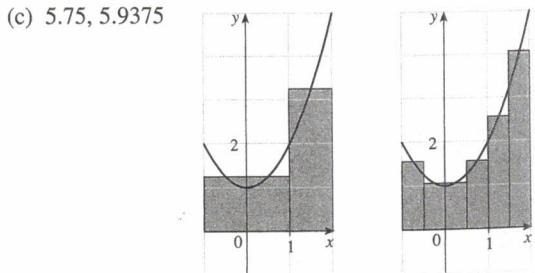
- 3.** (a) 0.7908, underestimate (b) 1.1835, overestimate



- 5.** (a) 8, 6.875 (b) 5, 5.375



- (c)** 5.75, 5.9375



- (d)** M_6

7. 0.2533, 0.2170, 0.2101, 0.2050; 0.2

9. (a) Left: 0.8100, 0.7937, 0.7904;
right: 0.7600, 0.7770, 0.7804

11. 34.7 ft, 44.8 ft 13. 63.2 L, 70 L 15. 155 ft

17. $\lim_{n \rightarrow \infty} \sum_{i=1}^n \sqrt[4]{1 + 15i/n} \cdot (15/n)$ 19. $\lim_{n \rightarrow \infty} \sum_{i=1}^n \left(\frac{i\pi}{2n} \cos \frac{i\pi}{2n} \right) \frac{\pi}{2n}$

21. The region under the graph of $y = \tan x$ from 0 to $\pi/4$

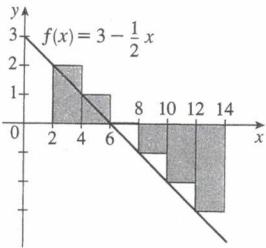
23. (a) $\lim_{n \rightarrow \infty} \frac{64}{n^6} \sum_{i=1}^n i^5$ (b) $\frac{n^2(n+1)^2(2n^2+2n-1)}{12}$ (c) $\frac{32}{3}$

25. $\sin b, 1$

EXERCISES 5.2 ■ PAGE 310

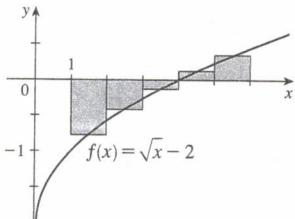
1. -6

The Riemann sum represents the sum of the areas of the two rectangles above the x -axis minus the sum of the areas of the three rectangles below the x -axis; that is, the net area of the rectangles with respect to the x -axis.



3. -0.856759

The Riemann sum represents the sum of the areas of the two rectangles above the x -axis minus the sum of the areas of the three rectangles below the x -axis.



5. (a) 4 (b) 6 (c) 10 7. -475, -85 9. 124.1644
11. 0.3084 13. 0.30843908, 0.30981629, 0.31015563

n	R_n
5	1.933766
10	1.983524
50	1.999342
100	1.999836

The values of R_n appear to be approaching 2.

17. $\int_2^6 \frac{1-x^2}{4+x^2} dx$ 19. $\int_1^8 \sqrt{2x+x^2} dx$ 21. 42
 23. $\frac{4}{3}$ 25. 3.75 29. $\lim_{n \rightarrow \infty} \sum_{i=1}^n \frac{2+4i/n}{1+(2+4i/n)^5} \cdot \frac{4}{n}$
 31. $\lim_{n \rightarrow \infty} \sum_{i=1}^n \left(\sin \frac{5\pi i}{n} \right) \frac{\pi}{n} = \frac{2}{5}$
 33. (a) 4 (b) 10 (c) -3 (d) 2 35. $-\frac{3}{4}$
 37. $3 + \frac{9}{4}\pi$ 39. 2.5 41. 0 43. 3 45. 22.5
 47. $\int_{-1}^5 f(x) dx$ 49. 122
 51. $2m \leq \int_0^2 f(x) dx \leq 2M$ by Comparison Property 8
 55. $3 \leq \int_1^4 \sqrt{x} dx \leq 6$ 57. $\frac{\pi}{12} \leq \int_{\pi/4}^{\pi/3} \tan x dx \leq \frac{\pi}{12} \sqrt{3}$
 59. $2 \leq \int_{-1}^1 \sqrt{1+x^4} dx \leq 2\sqrt{2}$ 69. $\int_0^1 x^4 dx$ 71. $\frac{1}{2}$

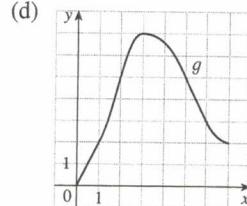
EXERCISES 5.3 ■ PAGE 321

1. One process undoes what the other one does. See the Fundamental Theorem of Calculus, page 320.

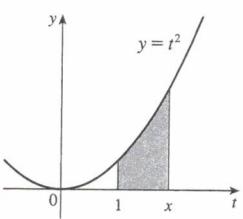
3. (a) 0, 2, 5, 7, 3

(b) (0, 3)

(c) $x = 3$



5.



(a), (b) x^2

7. $g'(x) = 1/(x^3 + 1)$

9. $g'(y) = y^2 \sin y$ 11. $F'(x) = -\sqrt{1 + \sec x}$

13. $h'(x) = -\sin^4(1/x)/x^2$ 15. $y' = \sqrt{\tan x} + \sqrt{\tan x} \sec^2 x$

17. $y' = \frac{3(1-3x)^3}{1+(1-3x)^2}$ 19. $\frac{3}{4}$ 21. 63

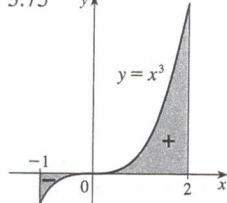
23. $\frac{5}{9}$ 25. $\frac{7}{8}$ 27. $\frac{156}{7}$ 29. $\frac{40}{3}$ 31. 1 33. $\frac{49}{3}$
35. 0

37. The function $f(x) = x^{-4}$ is not continuous on the interval $[-2, 1]$, so FTC2 cannot be applied.

39. The function $f(\theta) = \sec \theta \tan \theta$ is not continuous on the interval $[\pi/3, \pi]$, so FTC2 cannot be applied.

41. $\frac{243}{4}$ 43. 2

45. 3.75



47. $g'(x) = \frac{-2(4x^2-1)}{4x^2+1} + \frac{3(9x^2-1)}{9x^2+1}$
 49. $y' = 3x^{7/2} \sin(x^3) - \frac{\sin \sqrt{x}}{2\sqrt[4]{x}}$ 51. $\sqrt{257}$

53. (a) $-2\sqrt{n}, \sqrt{4n-2}$, n an integer > 0

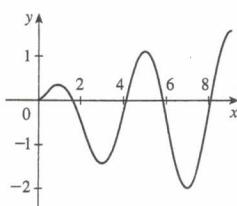
(b) $(0, 1), (-\sqrt{4n-1}, -\sqrt{4n-3})$, and $(\sqrt{4n-1}, \sqrt{4n+1})$, n an integer > 0 (c) 0.74

55. (a) Loc. max. at 1 and 5;
loc. min. at 3 and 7

(b) $x = 9$

(c) $(\frac{1}{2}, 2), (4, 6), (8, 9)$

(d) See graph at right.



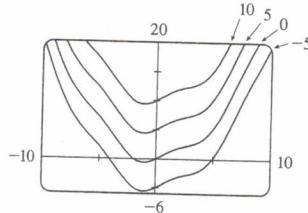
57. $\frac{1}{4}$ 65. $f(x) = x^{3/2}$, $a = 9$

67. (b) Average expenditure over $[0, t]$; minimize average expenditure

69. $\ln 3$ 71. π 73. $e^2 - 1$

EXERCISES 5.4 ■ PAGE 329

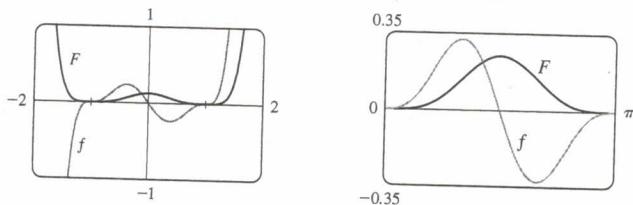
5. $\frac{1}{3}x^3 - (1/x) + C$ 7. $\frac{1}{5}x^5 - \frac{1}{8}x^4 + \frac{1}{8}x^2 - 2x + C$
 9. $2t - t^2 + \frac{1}{3}t^3 - \frac{1}{4}t^4 + C$ 11. $\frac{1}{3}x^3 - 4\sqrt{x} + C$
 13. $\frac{1}{2}\theta^2 + \csc \theta + C$ 15. $\tan \alpha + C$
 17. $\sin x + \frac{1}{4}x^2 + C$



19. 18 21. 231 23. 52 25. $\frac{256}{15}$ 27. $-\frac{63}{4}$ 29. $\frac{55}{63}$
 31. $2\sqrt{5}$ 33. 8 35. $1 + \pi/4$ 37. $\frac{256}{5}$ 39. 1
 41. -3.5 43. 0, 1.32; 0.84 45. $\frac{4}{3}$
 47. The increase in the child's weight (in pounds) between the ages of 5 and 10
 49. Number of gallons of oil leaked in the first 2 hours
 51. Increase in revenue when production is increased from 1000 to 5000 units
 53. Newton-meters (or joules) 55. (a) $-\frac{3}{2}$ m (b) $\frac{41}{6}$ m
 57. (a) $v(t) = \frac{1}{2}t^2 + 4t + 5$ m/s (b) $416\frac{2}{3}$ m
 59. $46\frac{2}{3}$ kg 61. 1.4 mi 63. \$58,000
 65. (b) At most 40%; $\frac{5}{36}$ 67. $-\cos x + \cosh x + C$
 69. $\frac{1}{3}x^3 + x + \tan^{-1}x + C$ 71. $\pi/6$

EXERCISES 5.5 ■ PAGE 338

1. $\frac{1}{3}\sin 3x + C$ 3. $\frac{2}{9}(x^3 + 1)^{3/2} + C$ 5. $-\frac{1}{4}\cos^4 \theta + C$
 7. $-\frac{1}{2}\cos(x^2) + C$ 9. $\frac{1}{63}(3x - 2)^{21} + C$
 11. $\frac{1}{3}(2x + x^2)^{3/2} + C$ 13. $-(1/\pi)\cos \pi t + C$
 15. $\frac{2}{3}\sqrt{3ax + bx^3} + C$ 17. $2\sin\sqrt{t} + C$ 19. $\frac{1}{7}\sin^7 \theta + C$
 21. $\frac{1}{2}(1 + z^3)^{2/3} + C$ 23. $-\frac{2}{3}(\cot x)^{3/2} + C$
 25. $\frac{1}{3}\sec^3 x + C$ 27. $-1/(\sin x) + C$
 29. $\frac{4}{7}(x + 2)^{7/4} - \frac{8}{3}(x + 2)^{3/4} + C$
 31. $\frac{1}{8}(x^2 - 1)^4 + C$ 33. $\frac{1}{4}\sin^4 x + C$



35. 0 37. $\frac{182}{9}$ 39. 4 41. 0 43. 3
 45. $\frac{1}{3}(2\sqrt{2} - 1)a^3$ 47. $\frac{16}{15}$ 49. $\frac{1}{2}(\sin 4 - \sin 1)$
 51. $\sqrt{3} - \frac{1}{3}$ 53. 6π 55. $\frac{5}{4\pi} \left(1 - \cos \frac{2\pi t}{5}\right) L$ 57. 5

65. $-\frac{1}{3}\ln|5 - 3x| + C$ 67. $\frac{1}{3}(\ln x)^3 + C$

69. $\frac{2}{3}(1 + e^x)^{3/2} + C$ 71. $e^{\tan x} + C$

73. $\tan^{-1}x + \frac{1}{2}\ln(1 + x^2) + C$ 75. $-\ln(1 + \cos^2 x) + C$

77. $\ln|\sin x| + C$ 79. 2 81. $\ln(e + 1)$ 83. $\pi^2/4$

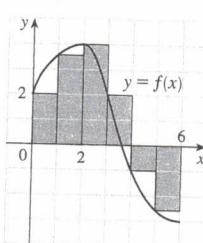
CHAPTER 5 REVIEW ■ PAGE 341

True-False Quiz

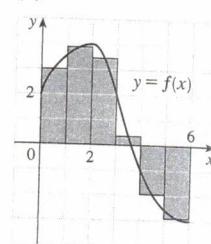
- I. True 3. True 5. False 7. True 9. True
 II. False 13. False 15. False

Exercises

I. (a) 8



(b) 5.7



3. $\frac{1}{2} + \pi/4$ 5. 3 7. f is c , f' is b , $\int_0^x f(t) dt$ is a
 9. 37 11. $\frac{9}{10}$ 13. -76 15. $\frac{21}{4}$ 17. Does not exist
 19. $\frac{1}{3}\sin 1$ 21. 0 23. $[(1/(2\pi))\sin^2 \pi t + C]$
 25. $\frac{1}{2}\sqrt{2} - \frac{1}{2}$ 27. $\frac{23}{3}$ 29. $2\sqrt{1 + \sin x} + C$ 31. $\frac{64}{5}$
 33. $F'(x) = x^2/(1 + x^3)$ 35. $g'(x) = 4x^3 \cos(x^8)$
 37. $y' = \frac{(2\cos x - \cos \sqrt{x})}{(2x)}$ 39. $4 \leq \int_1^3 \sqrt{x^2 + 3} dx \leq 4\sqrt{3}$

43. 0.280981

45. Number of barrels of oil consumed from Jan. 1, 2000, through Jan. 1, 2008

47. 72,400 49. 3 51. $(1 + x^2)(x \cos x + \sin x)/x^2$

PROBLEMS PLUS ■ PAGE 345

- I. $\pi/2$ 3. $f(x) = \frac{1}{2}x$ 5. -1 7. $[-1, 2]$
 9. (a) $\frac{1}{2}(n - 1)n$ (b) $\frac{1}{2}[b](2b - [b] - 1) - \frac{1}{2}[a](2a - [a] - 1)$
 15. $2(\sqrt{2} - 1)$

CHAPTER 6

EXERCISES 6.1 ■ PAGE 352

- I. $\frac{32}{3}$ 3. $\frac{4}{3}$ 5. 19.5 7. $\frac{1}{6}$ 9. $\frac{4}{3}$ 11. $\frac{1}{3}$ 13. 72

15. $6\sqrt{3}$ 17. $\frac{59}{12}$ 19. $\frac{32}{3}$ 21. $\frac{8}{3}$ 23. $\frac{1}{2}$

25. $2 - \pi/2$ 27. $\frac{3}{4}$ 29. 6.5 31. $\frac{3}{2}\sqrt{3} - 1$

33. 0.6407 35. 0, 0.90; 0.04 37. 8.38 39. $12\sqrt{6} - 9$

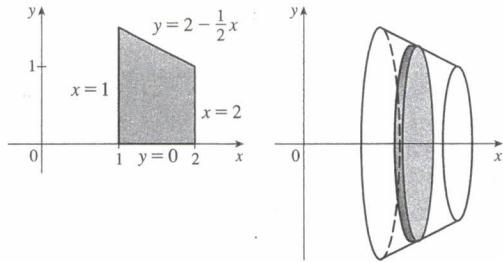
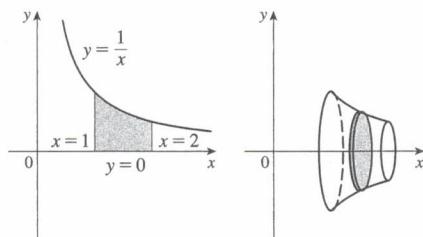
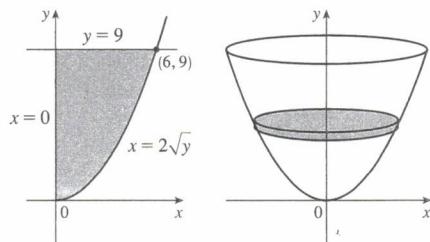
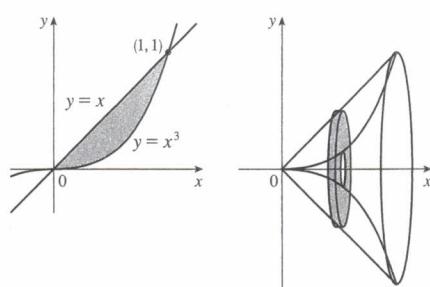
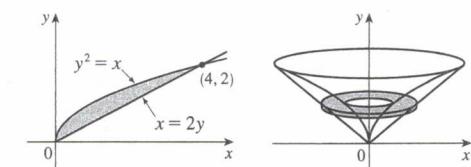
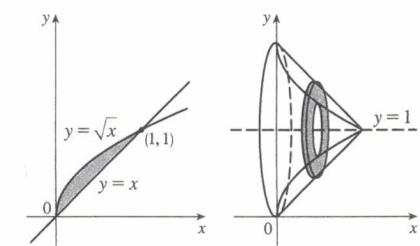
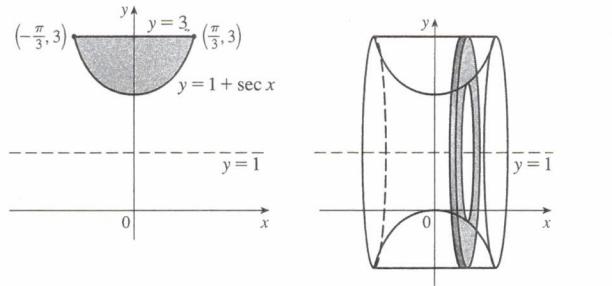
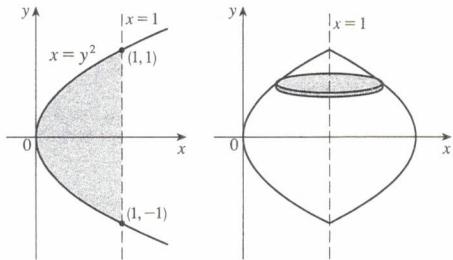
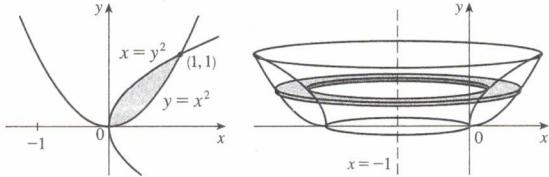
41. $117\frac{1}{3}$ ft 43. 4232 cm^2

45. (a) Car A (b) The distance by which A is ahead of B after 1 minute (c) Car A (d) $t \approx 2.2$ min

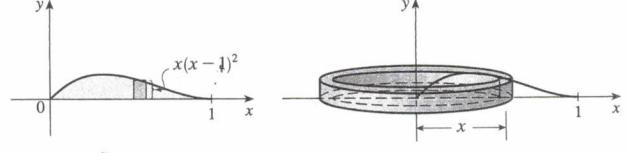
47. $\frac{24}{5}\sqrt{3}$ 49. $4^{2/3}$ 51. ± 6 53. $\ln 2 - \frac{1}{2}$

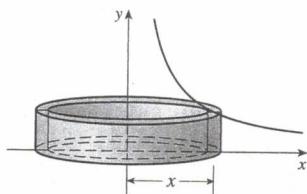
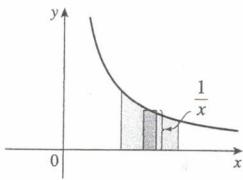
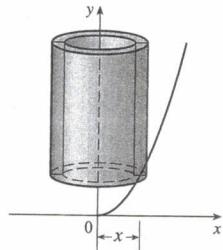
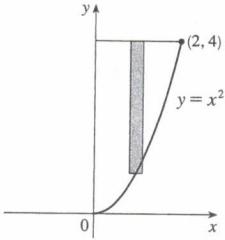
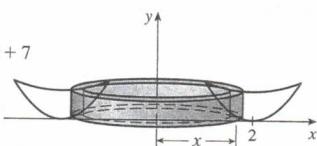
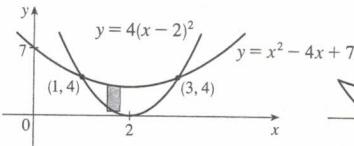
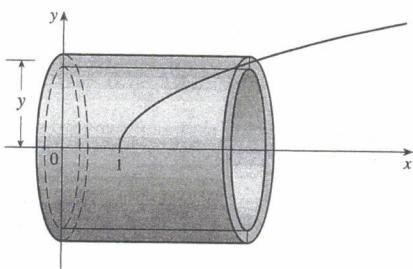
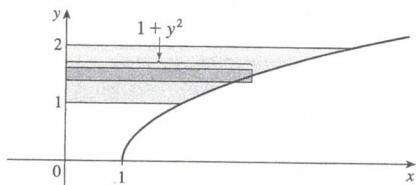
55. $2 - 2 \ln 2$

EXERCISES 6.2 ■ PAGE 362

1. $19\pi/12$ 3. $\pi/2$ 5. 162π 7. $4\pi/21$ 9. $64\pi/15$ 11. $\pi/6$ 13. $2\pi(\frac{4}{3}\pi - \sqrt{3})$ 15. $16\pi/15$ 17. $29\pi/30$ 19. $\pi/7$ 21. $\pi/10$ 23. $\pi/2$ 25. $7\pi/15$ 27. $5\pi/14$ 29. $13\pi/30$ 31. $\pi \int_0^{\pi/4} (1 - \tan^3 x)^2 dx$ 33. $\pi \int_0^{\pi} [1^2 - (1 - \sin x)^2] dx$ 35. $\pi \int_{-2\sqrt{2}}^{2\sqrt{2}} [5^2 - (\sqrt{1+y^2} + 2)^2] dy$ 37. $-1.288, 0.884; 23.780$ 39. $\frac{11}{8}\pi^2$ 41. Solid obtained by rotating the region $0 \leq y \leq \cos x$, $0 \leq x \leq \pi/2$ about the x-axis43. Solid obtained by rotating the region above the x-axis bounded by $x = y^2$ and $x = y^4$ about the y-axis45. 1110 cm^3 47. (a) 196 (b) 838 49. $\frac{1}{3}\pi r^2 h$ 51. $\pi h^2(r - \frac{1}{3}h)$ 53. $\frac{2}{3}b^2 h$ 55. 10 cm^3 57. 2459. $\frac{1}{3}$ 61. $\frac{8}{15}$ 63. (a) $8\pi R \int_0^r \sqrt{r^2 - y^2} dy$ (b) $2\pi^2 r^2 R$ 65. (b) $\pi r^2 h$ 67. $\frac{5}{12}\pi r^3$ 69. $8 \int_0^r \sqrt{R^2 - y^2} \sqrt{r^2 - y^2} dy$

EXERCISES 6.3 ■ PAGE 368

1. Circumference = $2\pi x$, height = $x(x-1)^2$; $\pi/15$ 

3. 2π 5. 8π 7. 16π 9. $21\pi/2$ 11. $768\pi/7$ 13. $16\pi/3$ 15. $7\pi/15$ 17. $8\pi/3$ 19. $5\pi/14$ 21. $\int_{2\pi}^{3\pi} 2\pi x \sin x \, dx$ 23. $\int_0^1 2\pi(x+1)[\sin(\pi x/2) - x^4] \, dx$ 25. $\int_0^\pi 2\pi(4-y)\sqrt{\sin y} \, dy$ 27. 3.6829. Solid obtained by rotating the region $0 \leq y \leq x^4$, $0 \leq x \leq 3$ about the y-axis

31. Solid obtained by rotating the region bounded by

(i) $x = 1 - y^2$, $x = 0$, and $y = 0$, or (ii) $x = y^2$, $x = 1$, and $y = 0$ about the line $y = 3$ 33. 0, 1.32; 4.05 35. $\frac{1}{32}\pi r^3$ 37. 8π 39. $\frac{63}{2}\pi$ 41. $\frac{4}{3}\pi$ 43. $\frac{4}{3}\pi r^3$ 45. $\frac{1}{3}\pi r^2 h$

EXERCISES 6.4 ■ PAGE 373

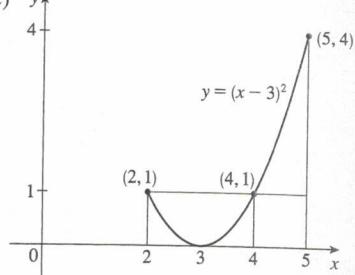
1. 588 J 3. 9 ft-lb 5. 180 J 7. $\frac{15}{4}$ ft-lb
 9. (a) $\frac{25}{24} \approx 1.04$ J (b) 10.8 cm 11. $W_2 = 3W_1$
 13. (a) 625 ft-lb (b) $\frac{1875}{4}$ ft-lb 15. 650,000 ft-lb
 17. 3857 J 19. 2450 J 21. $\approx 1.06 \times 10^6$ J

23. $\approx 1.04 \times 10^5$ ft-lb 25. 2.0 m 29. $Gm_1m_2\left(\frac{1}{a} - \frac{1}{b}\right)$

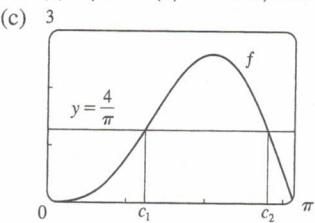
EXERCISES 6.5 ■ PAGE 377

- 1.
- $\frac{8}{3}$
- 3.
- $\frac{45}{28}$
- 5.
- $\frac{1}{15}(26^{3/2} - 1)$
- 7.
- $2/(5\pi)$

9. (a) 1 (b) 2, 4 (c)



11. (a)
- $4/\pi$
- (b)
- $\approx 1.24, 2.81$



- 15.
- $38\frac{1}{3}$
- 17.
- $(50 + 28/\pi)^\circ F \approx 59^\circ F$
19. 6 kg/m

- 21.
- $5/(4\pi) \approx 0.4$
- L

CHAPTER 6 REVIEW ■ PAGE 378

Exercises

- 1.
- $\frac{8}{3}$
- 3.
- $\frac{7}{12}$
- 5.
- $\frac{4}{3} + 4/\pi$
- 7.
- $64\pi/15$
- 9.
- $1656\pi/5$

- 11.
- $\frac{4}{3}\pi(2ah + h^2)^{3/2}$
- 13.
- $\int_{-\pi/3}^{\pi/3} 2\pi(\pi/2 - x)(\cos^2 x - \frac{1}{4}) \, dx$

15. (a)
- $2\pi/15$
- (b)
- $\pi/6$
- (c)
- $8\pi/15$

17. (a) 0.38 (b) 0.87

19. Solid obtained by rotating the region
- $0 \leq y \leq \cos x$
- ,
- $0 \leq x \leq \pi/2$
- about the y-axis

21. Solid obtained by rotating the region
- $0 \leq x \leq \pi$
- ,
- $0 \leq y \leq 2 - \sin x$
- about the x-axis

23. 36 25.
- $\frac{128}{3}\sqrt{3} \text{ m}^3$
27. 3.2 J

29. (a)
- $8000\pi/3 \approx 8378 \text{ ft-lb}$
- (b) 2.1 ft 31.
- $f(x)$

PROBLEMS PLUS ■ PAGE 380

1. (a)
- $f(t) = 3t^2$
- (b)
- $f(x) = \sqrt{2x/\pi}$
- 3.
- $\frac{32}{27}$

5. (b) 0.2261 (c) 0.6736 m

- (d) (i)
- $1/(105\pi) \approx 0.003 \text{ in/s}$
- (ii)
- $370\pi/3 \text{ s} \approx 6.5 \text{ min}$

- 9.
- $y = \frac{32}{9}x^2$

11. (a)
- $V = \int_0^h \pi[f(y)]^2 \, dy$
- (c)
- $f(y) = \sqrt{kA/(\pi C)} y^{1/4}$

Advantage: the markings on the container are equally spaced.

- 13.
- $b = 2a$
- 15.
- $B = 16A$

CHAPTER 7**EXERCISES 7.1 ■ PAGE 391**

1. (a) See Definition 1.
 (b) It must pass the Horizontal Line Test.
 3. No 5. Yes 7. No 9. No 11. Yes 13. No
 15. No 17. 2 19. 4

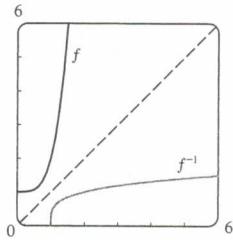
21. $F = \frac{9}{5}C + 32$; the Fahrenheit temperature as a function of the Celsius temperature; $[-273.15, \infty)$

23. $f^{-1}(x) = \frac{3}{2} - \frac{1}{2}x$ 25. $f^{-1}(x) = -\frac{1}{3}x^2 + \frac{10}{3}$, $x \geq 0$

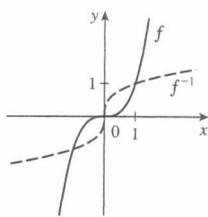
27. $y = \frac{(1-x)^2}{1+x}$, $-1 < x \leq 1$

29. $f^{-1}(x) = \sqrt[4]{x-1}$

31.



33. (b) $\frac{1}{12}$
 (c) $f^{-1}(x) = \sqrt[3]{x}$,
 domain = \mathbb{R} = range



37. $\frac{1}{7}$ 39. $2/\pi$ 41. $\frac{3}{2}$

43.

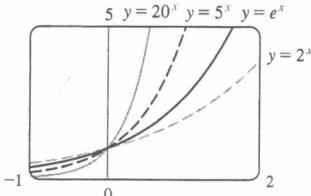
$f^{-1}(x) = -(\sqrt[3]{4}/6)(\sqrt[3]{D - 27x^2 + 20} - \sqrt[3]{D + 27x^2 - 20} + \sqrt[3]{2})$, where $D = 3\sqrt{3}\sqrt{27x^4 - 40x^2 + 16}$; two of the expressions are complex.

45. (a) $g^{-1}(x) = f^{-1}(x) - c$ (b) $h^{-1}(x) = (1/c)f^{-1}(x)$

EXERCISES 7.2 ■ PAGE 402

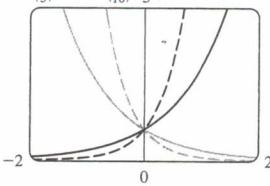
1. (a) $f(x) = a^x$, $a > 0$ (b) \mathbb{R} (c) $(0, \infty)$
 (d) See Figures 6(c), 6(b), and 6(a), respectively.

3.



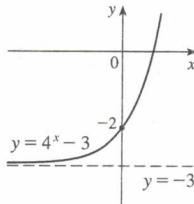
All approach 0 as $x \rightarrow -\infty$, all pass through $(0, 1)$, and all are increasing. The larger the base, the faster the rate of increase.

5. $y = (\frac{1}{3})^x$ $y = (\frac{1}{10})^x$ $y = 10^x$ $y = 3^x$

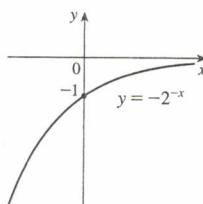


The functions with base greater than 1 are increasing and those with base less than 1 are decreasing. The latter are reflections of the former about the y-axis.

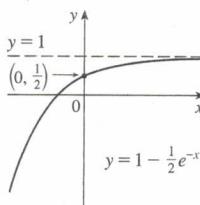
7.



9.



11.



13. (a) $y = e^x - 2$ (b) $y = e^{x-2}$ (c) $y = -e^x$
 (d) $y = e^{-x}$ (e) $y = -e^{-x}$

15. (a) $(-\infty, \infty)$ (b) $(-\infty, 0) \cup (0, \infty)$

17. $f(x) = 3 \cdot 2^x$ 21. At $x \approx 35.8$ 23. ∞ 25. 1

27. 0 29. 0 31. $f'(x) = e^x(x^3 + 3x^2 + 2x + 2)$

33. $y' = 3ax^2e^{ax^3}$ 35. $f'(u) = (-1/u^2)e^{1/u}$

37. $F'(t) = e^{t \sin 2t}(2t \cos 2t + \sin 2t)$

39. $y' = 3e^{3x}/\sqrt{1+2e^{3x}}$ 41. $y' = e^{e^x}e^x$

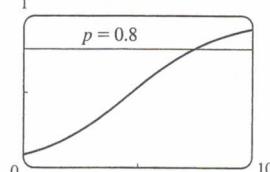
43. $y' = \frac{(ad-bc)e^x}{(ce^x+d)^2}$ 45. $y' = \frac{4e^{2x}}{(1+e^{2x})^2} \sin \frac{1-e^{2x}}{1+e^{2x}}$

47. $y = 2x + 1$ 49. $y' = (1-2xye^{x^2y})/(x^2e^{x^2y}-1)$

53. $-4, -2$ 55. $f^{(n)}(x) = 2^n e^{2x}$ 57. (b) -0.567143

59. (a) 1 (b) $kae^{-kt}/(1+ae^{-kt})^2$

(c) 1 $t \approx 7.4$ h



61. -1 63. $f(2) = 2/\sqrt{e}$, $f(-1) = -1/\sqrt[3]{e}$

65. (a) Inc. on $(2, \infty)$; dec. on $(-\infty, 2)$

(b) CU on $(-\infty, 3)$; CD on $(3, \infty)$ (c) $(3, -2e^{-3})$

67. A. $\{x | x \neq -1\}$

B. y-int. $1/e$ C. None

D. HA $y = 1$; VA $x = -1$

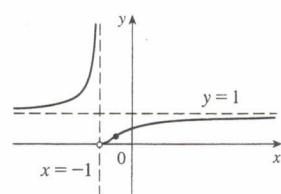
E. Inc. on $(-\infty, -1)$, $(-1, \infty)$

F. None

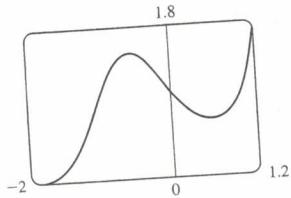
G. CU on $(-\infty, -1)$, $(-1, -\frac{1}{2})$

CD on $(-\frac{1}{2}, \infty)$; IP $(-\frac{1}{2}, 1/e^2)$

H. See graph at right



69. 28.57 min, when the rate of increase of drug level in the blood-stream is greatest; 85.71 min, when rate of decrease is greatest
 71. Loc. max. $f(-1/\sqrt{3}) = e^{2\sqrt{3}/9} \approx 1.5$;
 loc. min. $f(1/\sqrt{3}) = e^{-2\sqrt{3}/9} \approx 0.7$;
 IP $(-0.15, 1.15), (-1.09, 0.82)$



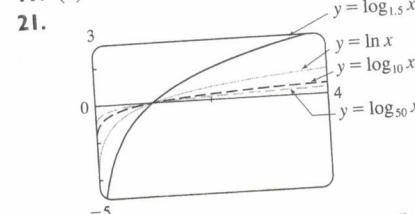
$$\begin{aligned} 73. \frac{1}{3}(1 - e^{-15}) & \quad 75. \frac{2}{3}(1 + e^x)^{3/2} + C \\ 77. \frac{1}{2}e^{2x} + 2x - \frac{1}{2}e^{-2x} + C & \quad 79. -e^{\cos x} + C \\ 81. 2e^{\sqrt{x}} + C & \quad 83. 4.644 \quad 85. \pi(e^2 - 1)/2 \quad 89. \frac{1}{2} \end{aligned}$$

EXERCISES 7.3 ■ PAGE 409

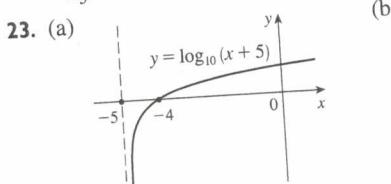
1. (a) It's defined as the inverse of the exponential function with base a , that is, $\log_a x = y \iff a^y = x$.
 (b) $(0, \infty)$ (c) \mathbb{R} (d) See Figure 1.
 3. (a) 3 (b) -3 5. (a) -2 (b) 15
 7. (a) 3 (b) -2
 9. $3 \log_2 x + \log_2 y - 2 \log_2 z$ 11. $10 \ln u + 10 \ln v$
 13. $\log_{10}(ac/b)$ 15. $\ln 1215$

$$17. \ln \frac{(1+x^2)\sqrt{x}}{\sin x}$$

$$19. (a) 0.402430 (b) 1.454240 (c) 1.651496$$



All graphs approach $-\infty$ as $x \rightarrow 0^+$, all pass through $(1, 0)$, and all are increasing. The larger the base, the slower the rate of increase.



$$25. (a) \sqrt{e} \quad (b) -\ln 5$$

$$27. (a) 5 + \log_2 3 \text{ or } 5 + (\ln 3)/\ln 2 \quad (b) \frac{1}{2}(1 + \sqrt{1 + 4e})$$

$$29. -3, 0 \quad 31. e^e \quad 33. \ln 3$$

$$35. (a) 0.5210 \quad (b) 3.0949$$

$$37. (a) x < \ln 10 \quad (b) x > 1/e$$

$$39. \text{About 1,084,588 mi} \quad 41. 8.3$$

43. (a) $f^{-1}(n) = (3/\ln 2) \ln(n/100)$; the time elapsed when there

are n bacteria (b) After about 26.9 hours

$$45. -\infty \quad 47. 0 \quad 49. \infty \quad 51. (-\infty, -3) \cup (3, \infty)$$

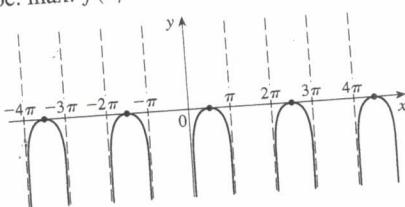
$$53. (a) (-\infty, \frac{1}{2}\ln 3] \quad (b) f^{-1}(x) = \frac{1}{2}\ln(3 - x^2), [0, \sqrt{3}]$$

$$55. y = e^x - 3 \quad 57. f^{-1}(x) = \sqrt[3]{\ln x} \quad 59. y = \frac{1}{10^x - 1}$$

61. $(-\frac{1}{2}\ln 3, \infty)$ 63. $(-\infty, 0), (4, \infty)$
 65. (b) $f^{-1}(x) = \frac{1}{2}(e^x - e^{-x})$ 67. f is a constant function
 71. $-1 \leq x \leq 1 - \sqrt{3}$ or $1 + \sqrt{3} \leq x \leq 3$

EXERCISES 7.4 ■ PAGE 419

1. The differentiation formula is simplest.
3. $f'(x) = \frac{\cos(\ln x)}{x}$ 5. $f'(x) = \frac{3}{(3x - 1)\ln 2}$
 7. $f'(x) = \frac{1}{5x\sqrt[5]{(\ln x)^4}}$ 9. $f'(x) = \frac{\sin x}{x} + \cos x \ln(5x)$
 11. $F'(t) = \frac{6}{2t+1} - \frac{12}{3t-1}$ 13. $g'(x) = \frac{2x^2 - 1}{x(x^2 - 1)}$
 15. $f'(u) = \frac{1 + \ln 2}{u[1 + \ln(2u)]^2}$ 17. $h'(t) = 3t^2 - 3^t \ln 3$
 19. $y' = \frac{10x + 1}{5x^2 + x - 2}$ 21. $y' = \frac{-x}{1+x}$
 23. $y' = \frac{1}{\ln 10} + \log_{10} x$ 25. $y' = 5^{-1/x}(\ln 5)/x^2$
 27. $y' = x + 2x \ln(2x); y'' = 3 + 2 \ln(2x)$
 29. $y' = \frac{1}{\sqrt{1+x^2}}; y'' = \frac{-x}{(1+x^2)^{3/2}}$
 31. $f'(x) = \frac{2x-1-(x-1)\ln(x-1)}{(x-1)[1-\ln(x-1)]^2};$
 $(1, 1+e) \cup (1+e, \infty)$
 33. $f'(x) = \frac{2(x-1)}{x(x-2)}; (-\infty, 0) \cup (2, \infty)$
 35. $\frac{1}{2} \quad 37. y = 3x - 2 \quad 39. \cos x + 1/x$
 41. $y' = (2x+1)^5(x^4-3)^6 \left(\frac{10}{2x+1} + \frac{24x^3}{x^4-3} \right)$
 43. $y' = \frac{\sin^2 x \tan^4 x}{(x^2+1)^2} \left(2 \cot x + \frac{4 \sec^2 x}{\tan x} - \frac{4x}{x^2+1} \right)$
 45. $y' = x^x(1 + \ln x)$
 47. $y' = x^{\sin x} \left(\frac{\sin x}{x} + \cos x \ln x \right)$
 49. $y' = (\cos x)^x(-x \tan x + \ln \cos x)$
 51. $y' = (\tan x)^{1/x} \left(\frac{\sec^2 x}{x \tan x} - \frac{\ln \tan x}{x^2} \right)$
 53. $y' = \frac{2x}{x^2 + y^2 - 2y}$ 55. $f^{(n)}(x) = \frac{(-1)^{n-1}(n-1)!}{(x-1)^n}$
 57. 2.958516, 5.290718
 59. CU on $(e^{8/3}, \infty)$, CD on $(0, e^{8/3})$, IP $(e^{8/3}, \frac{8}{3}e^{-4/3})$
 61. A. All x in $(2n\pi, (2n+1)\pi)$ (n an integer)
 B. x -int. $\pi/2 + 2n\pi$ C. Period 2π D. VA $x = n\pi$
 E. Inc. on $(2n\pi, \pi/2 + 2n\pi)$; dec. on $(\pi/2 + 2n\pi, (2n+1)\pi)$
 F. Loc. max. $f(\pi/2 + 2n\pi) = 0$ G. CD on $(2n\pi, (2n+1)\pi)$
 H.



63. A. \mathbb{R} B. y-int 0; x-int. 0

C. About y-axis D. None

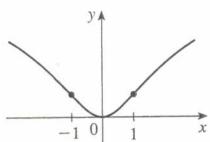
E. Inc. on $(0, \infty)$;

dec. on $(-\infty, 0)$

F. Loc. min. $f(0) = 0$

G. CU on $(-1, 1)$; CD on

$(-\infty, -1), (1, \infty)$;

IP $(\pm 1, \ln 2)$ H. See graph at right.

65. Inc. on $(0, 2.7), (4.5, 8.2), (10.9, 14.3)$;

IP $(3.8, 1.7), (5.7, 2.1), (10.0, 2.7), (12.0, 2.9)$

67. (a) $y = ab^t$ where $a \approx 100.01244$ and $b \approx 0.000045146$

(b) $-670.63 \mu\text{A}$

69. $3 \ln 2$ 71. $\frac{1}{3} \ln \frac{5}{2}$ 73. $\frac{1}{2}e^2 + e - \frac{1}{2}$

75. $\frac{1}{3}(\ln x)^3 + C$ 77. $-\ln(1 + \cos^2 x) + C$ 79. $90/(\ln 10)$

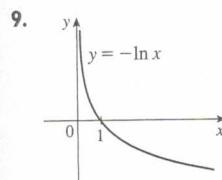
83. $\pi \ln 2$ 85. 45,974 J 87. $\frac{1}{3}$

89. $0 < m < 1, m - 1 - \ln m$

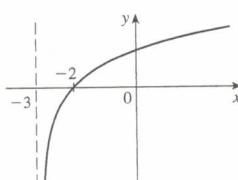
EXERCISES 7.2* ■ PAGE 428

1. $2 \ln r - \ln 3 - \frac{1}{2} \ln s$ 3. $10 \ln u + 10 \ln v$

5. $\ln 1215$ 7. $\ln \frac{(1+x^2)\sqrt{x}}{\sin x}$



11.



13. $-\infty$ 15. $f'(x) = (2 + \ln x)/(2\sqrt{x})$

17. $f'(x) = \frac{\cos(\ln x)}{x}$ 19. $f'(x) = \frac{1}{5x\sqrt[5]{(\ln x)^4}}$

21. $f'(x) = \frac{\sin x}{x} + \cos x \ln(5x)$ 23. $g'(x) = -2a/(a^2 - x^2)$

25. $F'(t) = \frac{6}{2t+1} - \frac{12}{3t-1}$ 27. $g'(x) = \frac{2x^2-1}{x(x^2-1)}$

29. $f'(u) = \frac{1+\ln 2}{u[1+\ln(2u)]^2}$ 31. $y' = \frac{10x+1}{5x^2+x-2}$

33. $y' = \sec^2(\ln(ax+b)) \frac{a}{ax+b}$

35. $y' = x + 2x \ln(2x); y'' = 3 + 2 \ln(2x)$

37. $f'(x) = \frac{2x-1-(x-1)\ln(x-1)}{(x-1)[1-\ln(x-1)]^2}; (1, 1+e) \cup (1+e, \infty)$

39. $f'(x) = -\frac{1}{2x\sqrt{1-\ln x}}; (0, e]$ 41. $\frac{1}{2}$ 43. $\cos x + 1/x$

45. $y = 2x-2$ 47. $y' = \frac{2x}{x^2+y^2-2y}$

49. $f^{(n)}(x) = \frac{(-1)^{n-1}(n-1)!}{(x-1)^n}$ 51. 2.958516, 5.290718

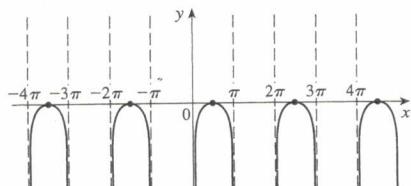
53. A. All x in $(2n\pi, (2n+1)\pi)$ (n an integer)

B. x-int. $\pi/2 + 2n\pi$ C. Period 2π D. VA $x = n\pi$

E. Inc. on $(2n\pi, \pi/2 + 2n\pi)$; dec. on $(\pi/2 + 2n\pi, (2n+1)\pi)$

F. Loc. max. $f(\pi/2 + 2n\pi) = 0$ G. CD on $(2n\pi, (2n+1)\pi)$

H.



55. A. \mathbb{R} B. y-int 0; x-int. 0

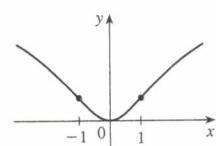
C. About y-axis D. None

E. Inc. on $(0, \infty)$;

dec. on $(-\infty, 0)$

F. Loc. min. $f(0) = 0$

G. CU on $(-1, 1)$; CD on $(-\infty, -1), (1, \infty)$;

IP $(\pm 1, \ln 2)$ H. See graph at right.

57. Inc. on $(0, 2.7), (4.5, 8.2), (10.9, 14.3)$;

IP $(3.8, 1.7), (5.7, 2.1), (10.0, 2.7), (12.0, 2.9)$

59. $y' = (2x+1)^5(x^4-3)^6 \left(\frac{10}{2x+1} + \frac{24x^3}{x^4-3} \right)$

61. $y' = \frac{\sin^2 x \tan^4 x}{(x^2+1)^2} \left(2 \cot x + \frac{4 \sec^2 x}{\tan x} - \frac{4x}{x^2+1} \right)$

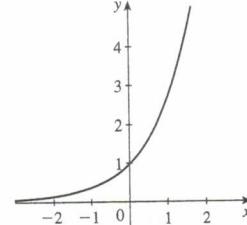
63. $3 \ln 2$ 65. $\frac{1}{3} \ln \frac{5}{2}$ 67. $\frac{1}{2}e^2 + e - \frac{1}{2}$ 69. $\frac{1}{3}(\ln x)^3 + C$

71. $-\ln(1 + \cos^2 x) + C$ 75. $\pi \ln 2$ 77. 45,974 J

79. $\frac{1}{3}$ 81. (b) 0.405 85. $0 < m < 1, m - 1 - \ln m$

EXERCISES 7.3* ■ PAGE 435

I.

If $f(x) = e^x$, then $f'(0) = 1$.

3. (a) $\frac{1}{25}$ (b) 10 5. (a) \sqrt{e} (b) $-\ln 5$

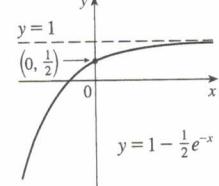
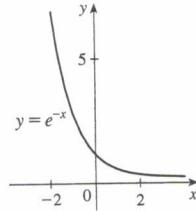
7. (a) $\frac{1}{3}(\ln k - 1)$ (b) $\frac{1}{2}(1 + \sqrt{1 + 4e})$

9. $-3, 0$ 11. $\ln 3$

13. (a) 0.5210 (b) 3.0949

15. (a) $x < \ln 10$ (b) $x > 1/e$

17.



19. (a) $(-\infty, \frac{1}{2} \ln 3]$ (b) $f^{-1}(x) = \frac{1}{2} \ln(3 - x^2), [0, \sqrt{3}]$

21. $y = e^x - 3$ 25. $f^{-1}(x) = \sqrt[3]{\ln x}$ 27. 1 29. 0

31. 0 33. $f'(x) = e^x(x^3 + 3x^2 + 2x + 2)$

35. $y' = 3ax^2e^{ax^3}$ 37. $f'(u) = (-1/u^2)e^{1/u}$

39. $F'(t) = e^{t \sin 2t}(2t \cos 2t + \sin 2t)$

41. $y' = 3e^{3x}/\sqrt{1+2e^{3x}}$

45. $y' = \frac{(ad-bc)e^x}{(ce^x+d)^2}$

49. $y = 2x + 1$

55. $-4, -2$

61. (a) 1

(c) 1

43. $y' = e^{e^x}e^x$

47. $y' = \frac{4e^{2x}}{(1+e^{2x})^2} \sin \frac{1-e^{2x}}{1+e^{2x}}$

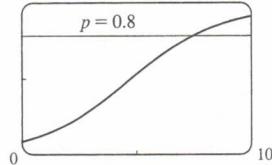
51. $y' = (1-2xye^{x^2y})/(x^2e^{x^2y}-1)$

57. $f^{(n)}(x) = 2^n e^{2x}$

59. (b) -0.567143

61. (b) $kae^{-kt}/(1+ae^{-kt})^2$

(t) $t \approx 7.4$ h



63. -1

65. $f(2) = 2/\sqrt{e}, f(-1) = -1/\sqrt[8]{e}$

67. (a) Inc. on $(2, \infty)$; dec. on $(-\infty, 2)$

(b) CU on $(-\infty, 3)$; CD on $(3, \infty)$

(c) $(3, -2e^{-3})$

69. A. $\{x | x \neq -1\}$

B. y-int. $1/e$

C. None

D. HA $y = 1$; VA $x = -1$

E. Inc. on $(-\infty, -1), (-1, \infty)$

F. None

G. CU on $(-\infty, -1), (-1, -\frac{1}{2})$

CD on $(-\frac{1}{2}, \infty)$; IP $(-\frac{1}{2}, 1/e^2)$

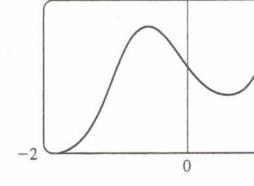
H. See graph at right

71. 28.57 min, when the rate of increase of drug level in the blood-stream is greatest; 85.71 min, when rate of decrease is greatest

73. Loc. max. $f(-1/\sqrt{3}) = e^{2\sqrt{3}/9} \approx 1.5$;

loc. min. $f(1/\sqrt{3}) = e^{-2\sqrt{3}/9} \approx 0.7$;

IP $(-0.15, 1.15), (-1.09, 0.82)$



75. $\frac{1}{3}(1 - e^{-15})$

77. $\frac{2}{3}(1 + e^x)^{3/2} + C$

79. $\frac{1}{2}e^{2x} + 2x - \frac{1}{2}e^{-2x} + C$

83. $2e^{\sqrt{x}} + C$

85. 4.644

87. $\pi(e^2 - 1)/2$

91. $\frac{1}{2}$

EXERCISES 7.4* ■ PAGE 445

I. (a) $a^x = e^{x \ln a}$

(b) $(-\infty, \infty)$

(c) $(0, \infty)$

(d) See Figures 1, 3, and 2.

3. $e^{\sqrt{7} \ln 5}$

5. $e^{x \ln \cos x}$

7. (a) 3

(b) -3

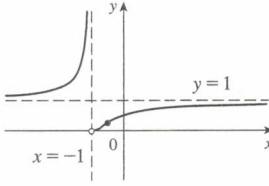
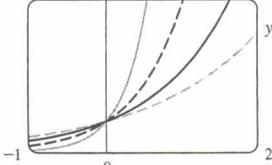
9. (a) 3

(b) -2

II. $y = 20^x$

$y = 5^x$

$y = e^x$



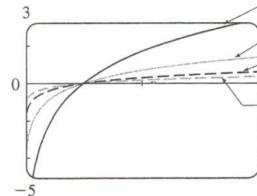
All approach 0 as $x \rightarrow -\infty$, all pass through $(0, 1)$, and all are increasing. The larger the base, the faster the rate of increase.

13. (a) 0.402430

(b) 1.454240

(c) 1.651496

15.



All graphs approach $-\infty$ as $x \rightarrow 0^+$, all pass through $(1, 0)$, and all are increasing. The larger the base, the slower the rate of increase.

17. $f(x) = 3 \cdot 2^x$

19. (b) About 1,084,588 mi

21. ∞

23. 0

25. $h'(t) = 3t^2 - 3^t \ln 3$

27. $y' = 5^{-1/x}(\ln 5)/x^2$

29. $f'(u) = 10 \ln 2 (2^u + 2^{-u})^9 (2^u - 2^{-u})$

31. $f'(x) = \frac{3}{(3x-1)\ln 2}$

33. $y' = \frac{1}{\ln 10} + \log_{10} x$

35. $y' = x^x(1 + \ln x)$

37. $y' = x^{\sin x} \left(\frac{\sin x}{x} + \cos x \ln x \right)$

39. $y' = (\cos x)^x (-x \tan x + \ln \cos x)$

41. $y' = (\tan x)^{1/x} \left(\frac{\sec^2 x}{x \tan x} - \frac{\ln \tan x}{x^2} \right)$

43. $y = (10 \ln 10)x + 10(1 - \ln 10)$

45. $90/(\ln 10)$

47. $(\ln x)^2/(2 \ln 10) + C$ [or $\frac{1}{2}(\ln 10)(\log_{10} x)^2 + C$]

49. $3^{\sin \theta} / \ln 3 + C$

51. $16/(5 \ln 5) - 1/(2 \ln 2)$

53. 0.600967

55. $y = \frac{1}{10^x - 1}$

57. 8.3

59. $10^8 / \ln 10$ dB/(watt/m²)

61. (a) $y = ab^t$ where $a \approx 100.01244$ and $b \approx 0.000045146$

(b) $-670.63 \mu\text{A}$

EXERCISES 7.5 ■ PAGE 453

I. About 235

3. (a) $100(4.2)^t$

(b) ≈ 7409

(c) $\approx 10,632$ bacteria/h

(d) $(\ln 100)/(\ln 4.2) \approx 3.2$ h

5. (a) 1508 million, 1871 million

(b) 2161 million

(c) 3972 million; wars in the first half of century, increased life

expectancy in second half

7. (a) $C e^{-0.0005t}$

(b) $-2000 \ln 0.9 \approx 211$ s

9. (a) $100 \times 2^{-t/30}$ mg

(b) ≈ 9.92 mg

(c) ≈ 199.3 years

11. ≈ 2500 years

13. (a) $\approx 137^\circ\text{F}$

(b) ≈ 116 min

15. (a) 13.3°C

(b) ≈ 67.74 min

17. (a) ≈ 64.5 kPa

(b) ≈ 39.9 kPa

19. (a) (i) \$3828.84

(ii) \$3840.25

(iii) \$3850.08

(iv) \$3851.61

(v) \$3852.01

(vi) \$3852.08

(b) $dA/dt = 0.05A, A(0) = 3000$

EXERCISES 7.6 ■ PAGE 461

1. (a) $\pi/3$

(b) π

3. (a) $\pi/4$

(b) $\pi/4$

5. (a) 10

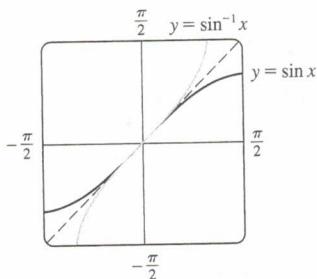
(b) $\pi/3$

7. $2/\sqrt{5}$

9. $\frac{2}{3}\sqrt{2}$

13. $x/\sqrt{1+x^2}$

15.



The second graph is the reflection of the first graph about the line $y = x$.

23. $y' = \frac{1}{2\sqrt{x}(1+x)}$

25. $y' = \frac{1}{\sqrt{-x^2-x}}$

27. $G'(x) = -1 - \frac{x \arccos x}{\sqrt{1-x^2}}$

29. $y' = -2e^{2x}/\sqrt{1-e^{4x}}$

31. $y' = -(\sin \theta)/(1+\cos^2 \theta)$

33. $h'(t) = 0$

35. $y' = \sqrt{a^2 - b^2}/(a + b \cos x)$

37. $g'(x) = 2/\sqrt{1-(3-2x)^2}; [1, 2], (1, 2)$

39. $\pi/6$

41. $1 - \frac{x \arcsin x}{\sqrt{1-x^2}}$

43. $-\pi/2$

45. $\pi/2$

47. At a distance $5 - 2\sqrt{5}$ from A

49. $\frac{1}{4}$ rad/s

51. A. $[-\frac{1}{2}, \infty)$

B. y-int. 0; x-int. 0

C. None

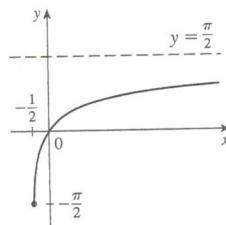
D. HA $y = \pi/2$

E. Inc. on $(-\frac{1}{2}, \infty)$

F. None

G. CD on $(-\frac{1}{2}, \infty)$

H. See graph at right



53. A. \mathbb{R}

B. y-int. 0; x-int. 0

C. About (0, 0)

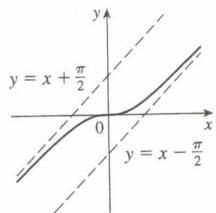
D. SA $y = x \pm \pi/2$

E. Inc. on \mathbb{R} F. None

G. CU on $(0, \infty)$; CD on $(-\infty, 0)$

IP (0, 0)

H. See graph at right.

55. Max. at $x = 0$, min. at $x \approx \pm 0.87$, IP at $x \approx \pm 0.52$

57. $\tan^{-1}x + x + C$

59. π 61. $\pi/12$

63. $\tan^{-1}x + \frac{1}{2}\ln(1+x^2) + C$

65. $\ln|\sin^{-1}x| + C$ 67. $\frac{1}{3}\sin^{-1}t^3 + C$

69. $2\tan^{-1}\sqrt{x} + C$ 73. $\pi/2 - 1$

EXERCISES 7.7 ■ PAGE 468

1. (a) 0 (b) 1 3. (a) $\frac{3}{4}$ (b) $\frac{1}{2}(e^2 - e^{-2}) \approx 3.62686$

5. (a) 1 (b) 0

21. $\operatorname{sech} x = \frac{3}{5}$, $\sinh x = \frac{4}{3}$, $\operatorname{csch} x = \frac{3}{4}$, $\tanh x = \frac{4}{5}$, $\coth x = \frac{5}{4}$

23. (a) 1 (b) -1 (c) ∞ (d) $-\infty$ (e) 0 (f) 1

(g) ∞ (h) $-\infty$ (i) 0

31. $f'(x) = x \cosh x$ 33. $h'(x) = \tanh x$

35. $y' = 3e^{\cosh 3x} \sinh 3x$ 37. $f'(t) = -2e^t \operatorname{sech}^2(e^t) \tanh(e^t)$

39. $y' = \frac{\operatorname{sech}^2 x}{1 + \tanh^2 x}$ 41. $G'(x) = \frac{-2 \sinh x}{(1 + \cosh x)^2}$

43. $y' = \frac{1}{2\sqrt{x}(1-x)}$ 45. $y' = \sinh^{-1}(x/3)$

47. $y' = \frac{-1}{x\sqrt{x^2+1}}$

51. (a) 0.3572 (b) 70.34°

53. (b) $y = 2 \sinh 3x - 4 \cosh 3x$

55. $(\ln(1 + \sqrt{2}), \sqrt{2})$

57. $\frac{1}{3}\cosh^3 x + C$ 59. $2 \cosh \sqrt{x} + C$ 61. $-\operatorname{csch} x + C$

63. $\ln\left(\frac{6+3\sqrt{3}}{4+\sqrt{7}}\right)$ 65. $\tanh^{-1}e^x + C$

67. (a) 0, 0.48 (b) 0.04

EXERCISES 7.8 ■ PAGE 478

1. (a) Indeterminate (b) 0 (c) 0
(d) ∞ , $-\infty$, or does not exist (e) Indeterminate

3. (a) $-\infty$ (b) Indeterminate (c) ∞

5. 2 7. $\frac{9}{5}$ 9. $-\infty$ 11. ∞ 13. p/q

15. 0 17. $-\infty$ 19. ∞ 21. $\frac{1}{2}$ 23. 1

25. $\ln\frac{5}{3}$ 27. 1 29. $\frac{1}{2}$ 31. 0 33. $-1/\pi^2$

35. $\frac{1}{2}a(a-1)$ 37. $\frac{1}{24}$ 39. π 41. 3 43. 0

45. $-2/\pi$ 47. $\frac{1}{2}$ 49. $\frac{1}{2}$ 51. ∞ 53. 1

55. e^{-2} 57. e^3 59. 1 61. e^4

63. $1/\sqrt{e}$ 65. e^2 67. $\frac{1}{4}$

69. A. \mathbb{R} B. y-int. 0; x-int. 0

C. None D. HA $y = 0$

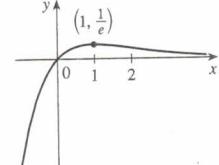
E. Inc. on $(-\infty, 1)$, dec. on $(1, \infty)$

F. Loc. max. $f(1) = 1/e$

G. CU on $(2, \infty)$; CD on $(-\infty, 2)$

IP $(2, 2/e^2)$

H. See graph at right.



71. A. \mathbb{R} B. y-int. 0; x-int. 0 C. About (0, 0) D. HA $y = 0$

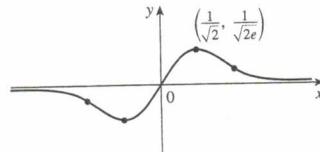
E. Inc. on $(-\sqrt{2}, 1/\sqrt{2})$; dec. on $(-\infty, -1/\sqrt{2})$, $(1/\sqrt{2}, \infty)$

F. Loc. min. $f(-1/\sqrt{2}) = -1/\sqrt{2e}$; loc. max. $f(1/\sqrt{2}) = 1/\sqrt{2e}$

G. CU on $(-\sqrt{3/2}, 0)$, $(\sqrt{3/2}, \infty)$; CD on $(-\infty, -\sqrt{3/2})$, $(0, \sqrt{3/2})$

IP $(\pm\sqrt{3/2}, \pm\sqrt{3/2}e^{-3/2})$, (0, 0)

H.



73. A. $(-1, \infty)$ B. y-int. 0; x-int. 0

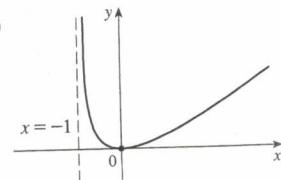
C. None D. VA $x = -1$

E. Inc. on $(0, \infty)$; dec. on $(-1, 0)$

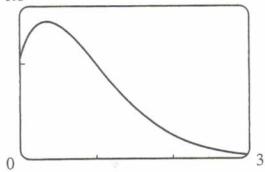
F. Loc. min. $f(0) = 0$

G. CU on $(-1, \infty)$

H. See graph at right.



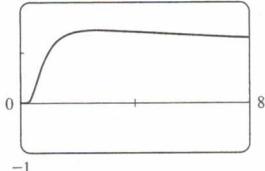
75. (a)



(b) $\lim_{x \rightarrow 0^+} x^{-x} = 1$

(c) Max. value $f(1/e) = e^{1/e} \approx 1.44$ (d) 1.0

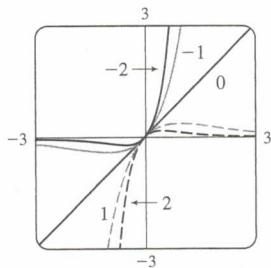
77. (a)



(b) $\lim_{x \rightarrow 0^+} x^{1/x} = 0$, $\lim_{x \rightarrow \infty} x^{1/x} = 1$

(c) Max. value $f(e) = e^{1/e} \approx 1.44$ (d) IP at $x \approx 0.58, 4.37$

79.

For $c > 0$, $\lim_{x \rightarrow \infty} f(x) = 0$ and $\lim_{x \rightarrow -\infty} f(x) = -\infty$.For $c < 0$, $\lim_{x \rightarrow \infty} f(x) = \infty$ and $\lim_{x \rightarrow -\infty} f(x) = 0$.As $|c|$ increases, the maximum and minimum points and the IPs get closer to the origin.

81. 1 87. $\pi/6$ 89. 56 97. $\frac{1}{3}$ 99. (a) 0

CHAPTER 7 REVIEW ■ PAGE 483

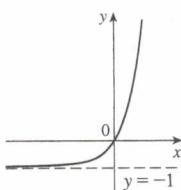
True-False Quiz

- I. True 3. False 5. True 7. True 9. False
 II. False 13. False 15. True 17. True

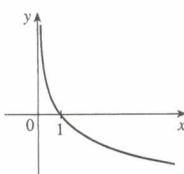
Exercises

1. No 3. (a) 7 (b) $\frac{1}{8}$

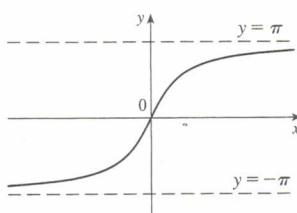
5.



7.



9.



11. (a) 9 (b) 2 13. $e^{1/3}$

15. $\ln \ln 17$ 17. $\sqrt{1+e}$

19. $\tan 1$ 21. $f'(t) = t + 2t \ln t$

23. $h'(\theta) = 2 \sec^2(2\theta)e^{\tan 2\theta}$ 25. $y' = 5 \sec 5x$

27. $y' = (1+c^2)e^{cx} \sin x$ 29. $y' = 2 \tan x$

31. $y' = -\frac{e^{1/x}(1+2x)}{x^4}$ 33. $y' = 3^{x \ln x} (\ln 3)(1+\ln x)$

35. $H'(v) = (v/(1+v^2)) + \tan^{-1} v$

37. $y' = 2x^2 \cosh(x^2) + \sinh(x^2)$

39. $y' = \cot x - \sin x \cos x$ 41. $y' = -(1/x)[1 + 1/(\ln x)^2]$

43. $y' = 3 \tanh 3x$ 45. $y' = (\cosh x)/\sqrt{\sinh^2 x - 1}$

47. $y' = \frac{-3 \sin(e^{\sqrt{\tan 3x}}) e^{\sqrt{\tan 3x}} \sec^2(3x)}{2\sqrt{\tan 3x}}$ 49. $f'(x) = g'(x)e^{g(x)}$

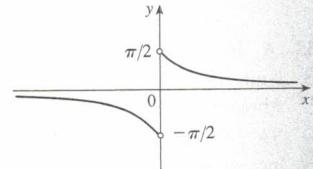
51. $g'(x)/g(x)$ 53. $2^x(\ln 2)^n$ 57. $y = -x + 2$

59. $(-3, 0)$ 61. (a) $y = \frac{1}{4}x + \frac{1}{4}(\ln 4 + 1)$ (b) $y = ex$

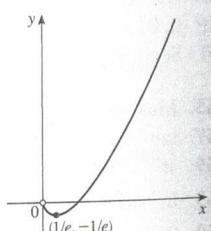
63. 0 65. 0 67. $-\infty$ 69. -1 71. π 73. 8

75. 0 77. $\frac{1}{2}$

79. A. $\{x \mid x \neq 0\}$ B. None
 C. About $(0, 0)$ D. HA $y = 0$
 E. Dec. on $(-\infty, 0)$ and $(0, \infty)$
 F. None
 G. CU on $(0, \infty)$; CD on $(-\infty, 0)$
 H. See graph at right.

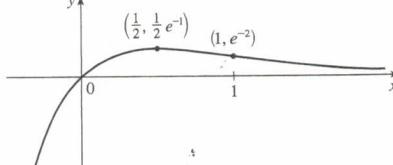


81. A. $(0, \infty)$ B. x -int. 1
 C. None D. None
 E. Inc. on $(1/e, \infty)$; dec. on $(0, 1/e)$
 F. Loc. min. $f(1/e) = -1/e$
 G. CU on $(0, \infty)$
 H. See graph at right.

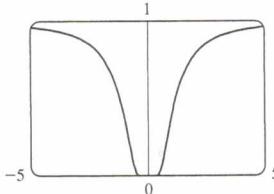


83. A. \mathbb{R} B. y-int. 0, x-int. 0 C. None D. HA $y = 0$
 E. Inc. on $(-\infty, \frac{1}{2})$, dec. on $(\frac{1}{2}, \infty)$ F. Loc. max. $f(\frac{1}{2}) = 1/(2e)$
 G. CU on $(1, \infty)$; CD on $(-\infty, 1)$; IP $(1, e^{-2})$

H.



85.



$$(\pm 0.82, 0.22); (\pm \sqrt{2/3}, e^{-3/2})$$

87. $v(t) = -Ae^{-ct}[c \cos(\omega t + \delta) + \omega \sin(\omega t + \delta)]$,
 $a(t) = Ae^{-ct}[(c^2 - \omega^2) \cos(\omega t + \delta) + 2\omega c \sin(\omega t + \delta)]$
89. (a) $200(3.24)^t$ (b) $\approx 22,040$
(c) $\approx 25,910$ bacteria/h (d) $(\ln 50)/(\ln 3.24) \approx 3.33$ h
91. 4.32 days 93. $\frac{1}{4}(1 - e^{-2})$ 95. $\arctan e - \pi/4$
97. $2e^{\sqrt{x}} + C$ 99. $\frac{1}{2}\ln|x^2 + 2x| + C$
101. $-\frac{1}{2}[\ln(\cos x)]^2 + C$ 103. $2^{\tan \theta}/\ln 2 + C$
105. $-(1/x) - 2 \ln|x| + x + C$ 109. $e^{\sqrt{x}}/(2x)$
111. $\frac{1}{3}\ln 4$ 113. $\pi^2/4$ 115. $\frac{2}{3}$ 117. $2/e$
121. $f(x) = e^{2x}(1 + 2x)/(1 - e^{-x})$

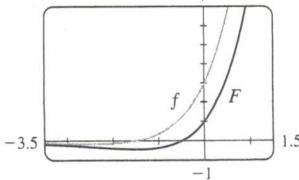
PROBLEMS PLUS ■ PAGE 487

7. $1/\sqrt{2}$ 9. $\frac{1}{2}$ 13. $2\sqrt{e}$ 15. $a \leq e^{1/e}$

CHAPTER 8

EXERCISES 8.1 ■ PAGE 493

1. $\frac{1}{3}x^3 \ln x - \frac{1}{9}x^3 + C$ 3. $\frac{1}{5}x \sin 5x + \frac{1}{25} \cos 5x + C$
5. $2(r-2)e^{r/2} + C$
7. $-\frac{1}{\pi}x^2 \cos \pi x + \frac{2}{\pi^2}x \sin \pi x + \frac{2}{\pi^3} \cos \pi x + C$
9. $\frac{1}{2}(2x+1) \ln(2x+1) - x + C$
11. $t \arctan 4t - \frac{1}{8} \ln(1+16t^2) + C$
13. $\frac{1}{2}t \tan 2t - \frac{1}{4} \ln|\sec 2t| + C$
15. $x(\ln x)^2 - 2x \ln x + 2x + C$
17. $\frac{1}{13}e^{2\theta}(2 \sin 3\theta - 3 \cos 3\theta) + C$
19. $\pi/3$ 21. $1 - 1/e$ 23. $\frac{1}{2} - \frac{1}{2} \ln 2$ 25. $\frac{1}{4} - \frac{3}{4}e^{-2}$
27. $\frac{1}{6}(\pi + 6 - 3\sqrt{3})$ 29. $\sin x(\ln \sin x - 1) + C$
31. $\frac{32}{5}(\ln 2)^2 - \frac{64}{25} \ln 2 + \frac{62}{125}$
33. $2\sqrt{x} \sin \sqrt{x} + 2 \cos \sqrt{x} + C$ 35. $-\frac{1}{2} - \pi/4$
37. $\frac{1}{2}(x^2 - 1) \ln(1+x) - \frac{1}{4}x^2 + \frac{1}{2}x + \frac{3}{4} + C$
39. $(2x+1)e^x + C$

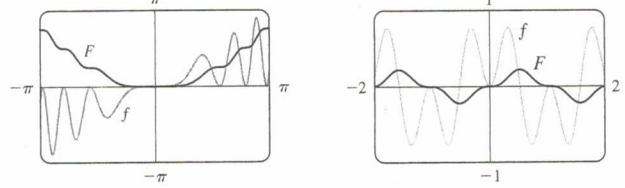


41. $\frac{1}{3}x^2(1+x^2)^{3/2} - \frac{2}{15}(1+x^2)^{5/2} + C$
43. (b) $-\frac{1}{4} \cos x \sin^2 x + \frac{3}{8}x - \frac{3}{16} \sin 2x + C$
45. (b) $\frac{2}{3}, \frac{8}{15}$ 51. $x(\ln x)^3 - 3x(\ln x)^2 + 6x \ln x - 6x + C$
53. $\frac{25}{4} - \frac{75}{4}e^{-2}$ 55. 1.0475, 2.8731; 2.1828 57. $4 - 8/\pi$

59. $2\pi e$ 61. $\frac{9}{2} \ln 3 - \frac{13}{9}$ 63. $2 - e^{-t}(t^2 + 2t + 2) m$
65. 2

EXERCISES 8.2 ■ PAGE 501

1. $\frac{1}{5} \cos^5 x - \frac{1}{3} \cos^3 x + C$ 3. $-\frac{11}{384}$
5. $\frac{1}{3\pi} \sin^3(\pi x) - \frac{2}{5\pi} \sin^5(\pi x) + \frac{1}{7\pi} \sin^7(\pi x) + C$
7. $\pi/4$ 9. $3\pi/8$ 11. $\frac{3}{2}\theta + 2 \sin \theta + \frac{1}{4} \sin 2\theta + C$
13. $\pi/16$ 15. $\frac{2}{45}\sqrt{\sin \alpha}(45 - 18 \sin^2 \alpha + 15 \sin^4 \alpha) + C$
17. $\frac{1}{2} \cos^2 x - \ln|\cos x| + C$ 19. $\ln|\sin x| + 2 \sin x + C$
21. $\frac{1}{2} \tan^2 x + C$ 23. $\tan x - x + C$
25. $\frac{1}{5} \tan^5 t + \frac{2}{3} \tan^3 t + \tan t + C$ 27. $\frac{117}{8}$
29. $\frac{1}{3} \sec^3 x - \sec x + C$
31. $\frac{1}{4} \sec^4 x - \tan^2 x + \ln|\sec x| + C$
33. $\frac{1}{6} \tan^6 \theta + \frac{1}{4} \tan^4 \theta + C$
35. $x \sec x - \ln|\sec x + \tan x| + C$ 37. $\sqrt{3} - \frac{1}{3}\pi$
39. $\frac{1}{3} \csc^3 \alpha - \frac{1}{5} \csc^5 \alpha + C$ 41. $\ln|\csc x - \cot x| + C$
43. $-\frac{1}{6} \cos 3x - \frac{1}{26} \cos 13x + C$ 45. $\frac{1}{8} \sin 4\theta - \frac{1}{12} \sin 6\theta + C$
47. $\frac{1}{2} \sin 2x + C$ 49. $\frac{1}{10} \tan^5(t^2) + C$
51. $\frac{1}{4}x^2 - \frac{1}{4} \sin(x^2) \cos(x^2) + C$ 53. $\frac{1}{6} \sin 3x - \frac{1}{18} \sin 9x + C$



55. 0 57. 1 59. 0 61. $\pi^2/4$ 63. $\pi(2\sqrt{2} - \frac{5}{2})$
65. $s = (1 - \cos^3 \omega t)/(3\omega)$

EXERCISES 8.3 ■ PAGE 508

1. $\sqrt{x^2 - 9}/(9x) + C$ 3. $\frac{1}{3}(x^2 - 18)\sqrt{x^2 + 9} + C$
5. $\pi/24 + \sqrt{3}/8 - \frac{1}{4}$ 7. $-\sqrt{25 - x^2}/(25x) + C$
9. $\ln(\sqrt{x^2 + 16} + x) + C$ 11. $\frac{1}{4} \sin^{-1}(2x) + \frac{1}{2}x\sqrt{1 - 4x^2} + C$
13. $\frac{1}{6} \sec^{-1}(x/3) - \sqrt{x^2 - 9}/(2x^2) + C$
15. $\frac{1}{16}\pi a^4$ 17. $\sqrt{x^2 - 7} + C$
19. $\ln|(\sqrt{1+x^2} - 1)/x| + \sqrt{1+x^2} + C$ 21. $\frac{9}{500}\pi$
23. $\frac{9}{2} \sin^{-1}((x-2)/3) + \frac{1}{2}(x-2)\sqrt{5+4x-x^2} + C$
25. $\sqrt{x^2 + x + 1} - \frac{1}{2} \ln(\sqrt{x^2 + x + 1} + x + \frac{1}{2}) + C$
27. $\frac{1}{2}(x+1)\sqrt{x^2 + 2x} - \frac{1}{2} \ln|x+1 + \sqrt{x^2 + 2x}| + C$
29. $\frac{1}{4} \sin^{-1}(x^2) + \frac{1}{4}x^2\sqrt{1-x^4} + C$
33. $\frac{1}{6}(\sqrt{48} - \sec^{-1} 7)$ 37. 0.81, 2; 2.10
41. $r\sqrt{R^2 - r^2} + \pi r^2/2 - R^2 \arcsin(r/R)$ 43. $2\pi^2 R r^2$

EXERCISES 8.4 ■ PAGE 517

1. (a) $\frac{A}{x+3} + \frac{B}{3x+1}$ (b) $\frac{A}{x} + \frac{B}{x+1} + \frac{C}{(x+1)^2}$
3. (a) $\frac{A}{x} + \frac{B}{x^2} + \frac{C}{x^3} + \frac{Dx+E}{x^2+4}$
- (b) $\frac{A}{x+3} + \frac{B}{(x+3)^2} + \frac{C}{x-3} + \frac{D}{(x-3)^2}$

5. (a) $1 + \frac{A}{x-1} + \frac{B}{x+1} + \frac{Cx+D}{x^2+1}$

(b) $\frac{At+B}{t^2+1} + \frac{Ct+D}{t^2+4} + \frac{Et+F}{(t^2+4)^2}$

7. $x + 6 \ln|x-6| + C$

9. $2 \ln|x+5| - \ln|x-2| + C$

11. $\frac{1}{2} \ln \frac{3}{2}$

13. $a \ln|x-b| + C$

15. $\frac{7}{6} + \ln \frac{2}{3}$

17. $\frac{27}{5} \ln 2 - \frac{9}{5} \ln 3$ (or $\frac{9}{5} \ln \frac{8}{3}$)

19. $-\frac{1}{36} \ln|x+5| + \frac{1}{6} \frac{1}{x+5} + \frac{1}{36} \ln|x-1| + C$

21. $\frac{1}{2}x^2 - 2 \ln(x^2+4) + 2 \tan^{-1}(x/2) + C$

23. $2 \ln|x| + (1/x) + 3 \ln|x+2| + C$

25. $\ln|x-1| - \frac{1}{2} \ln(x^2+9) - \frac{1}{3} \tan^{-1}(x/3) + C$

27. $\frac{1}{2} \ln(x^2+1) + (1/\sqrt{2}) \tan^{-1}(x/\sqrt{2}) + C$

29. $\frac{1}{2} \ln(x^2+2x+5) + \frac{3}{2} \tan^{-1}\left(\frac{x+1}{2}\right) + C$

31. $\frac{1}{3} \ln|x-1| - \frac{1}{6} \ln(x^2+x+1) - \frac{1}{\sqrt{3}} \tan^{-1}\frac{2x+1}{\sqrt{3}} + C$

33. $\frac{1}{4} \ln \frac{8}{3}$

35. $\frac{1}{16} \ln|x| - \frac{1}{32} \ln(x^2+4) + \frac{1}{8(x^2+4)} + C$

37. $\frac{7}{8} \sqrt{2} \tan^{-1}\left(\frac{x-2}{\sqrt{2}}\right) + \frac{3x-8}{4(x^2-4x+6)} + C$

39. $\ln \left| \frac{\sqrt{x+1}-1}{\sqrt{x+1}+1} \right| + C$

41. $2 + \ln \frac{25}{9}$

43. $\frac{3}{10}(x^2+1)^{5/3} - \frac{3}{4}(x^2+1)^{2/3} + C$

45. $2\sqrt{x} + 3\sqrt[3]{x} + 6\sqrt[6]{x} + 6 \ln|\sqrt[6]{x}-1| + C$

47. $\ln \left[\frac{(e^x+2)^2}{e^x+1} \right] + C$

49. $\ln|\tan t+1| - \ln|\tan t+2| + C$

51. $(x - \frac{1}{2}) \ln(x^2-x+2) - 2x + \sqrt{7} \tan^{-1}\left(\frac{2x-1}{\sqrt{7}}\right) + C$

53. $-\frac{1}{2} \ln 3 \approx -0.55$

55. $\frac{1}{2} \ln \left| \frac{x-2}{x} \right| + C$

61. $4 \ln \frac{2}{3} + 2$

63. $-1 + \frac{11}{3} \ln 2$

65. $t = -\ln P - \frac{1}{9} \ln(0.9P+900) + C$, where $C \approx 10.23$

67. (a) $\frac{24,110}{4879} \frac{1}{5x+2} - \frac{668}{323} \frac{1}{2x+1} - \frac{9438}{80,155} \frac{1}{3x-7} +$

$\frac{1}{260,015} \frac{22,098x+48,935}{x^2+x+5}$

(b) $\frac{4822}{4879} \ln|5x+2| - \frac{334}{323} \ln|2x+1| - \frac{3146}{80,155} \ln|3x-7| +$

$\frac{11,049}{260,015} \ln(x^2+x+5) + \frac{75,772}{260,015\sqrt{19}} \tan^{-1}\frac{2x+1}{\sqrt{19}} + C$

The CAS omits the absolute value signs and the constant of integration.

EXERCISES 8.5 ■ PAGE 524

1. $\sin x + \frac{1}{3} \sin^3 x + C$

3. $\sin x + \ln|\csc x - \cot x| + C$

5. $4 - \ln 9$

7. $e^{\pi/4} - e^{-\pi/4}$

9. $\frac{243}{5} \ln 3 - \frac{242}{25}$

11. $\frac{1}{2} \ln(x^2-4x+5) + \tan^{-1}(x-2) + C$

13. $\frac{1}{8} \cos^8 \theta - \frac{1}{6} \cos^6 \theta + C$ (or $\frac{1}{4} \sin^4 \theta - \frac{1}{3} \sin^6 \theta + \frac{1}{8} \sin^8 \theta + C$)

15. $x/\sqrt{1-x^2} + C$

17. $\frac{1}{4}x^2 - \frac{1}{2}x \sin x \cos x + \frac{1}{4} \sin^2 x + C$

(or $\frac{1}{4}x^2 - \frac{1}{4}x \sin 2x - \frac{1}{8} \cos 2x + C$)

19. $e^{e^x} + C$

21. $(x+1) \arctan \sqrt{x} - \sqrt{x} + C$

23. $\frac{4097}{45}$

25. $3x + \frac{23}{3} \ln|x-4| - \frac{5}{3} \ln|x+2| + C$

27. $x - \ln(1+e^x) + C$

29. $15 + 7 \ln \frac{2}{7}$

31. $\sin^{-1} x - \sqrt{1-x^2} + C$

33. $2 \sin^{-1}\left(\frac{x+1}{2}\right) + \frac{x+1}{2} \sqrt{3-2x-x^2} + C$

35. 0

37. $\pi/8 - \frac{1}{4}$

39. $\ln|\sec \theta - 1| - \ln|\sec \theta| + C$

41. $\theta \tan \theta - \frac{1}{2} \theta^2 - \ln|\sec \theta| + C$

43. $\frac{2}{3}(1+e^x)^{3/2} + C$

45. $-\frac{1}{3}(x^3+1)e^{-x^3} + C$

47. $\ln|x-1| - 3(x-1)^{-1} - \frac{3}{2}(x-1)^{-2} - \frac{1}{3}(x-1)^{-3} + C$

49. $\ln \left| \frac{\sqrt{4x+1}-1}{\sqrt{4x+1}+1} \right| + C$

51. $-\ln \left| \frac{\sqrt{4x^2+1}+1}{2x} \right| + C$

53. $\frac{1}{m} x^2 \cosh(mx) - \frac{2}{m^2} x \sinh(mx) + \frac{2}{m^3} \cosh(mx) + C$

55. $2 \ln \sqrt{x} - 2 \ln(1+\sqrt{x}) + C$

57. $\frac{3}{7}(x+c)^{7/3} - \frac{3}{4}c(x+c)^{4/3} + C$

59. $\sin(\sin x) - \frac{1}{3} \sin^3(\sin x) + C$

61. $2(x-2\sqrt{x}+2)e^{\sqrt{x}} + C$

63. $-\tan^{-1}(\cos^2 x) + C$

65. $\frac{2}{3}[(x+1)^{3/2} - x^{3/2}] + C$

67. $\sqrt{2} - 2/\sqrt{3} + \ln(2+\sqrt{3}) - \ln(1+\sqrt{2})$

69. $e^x - \ln(1+e^x) + C$

71. $-\sqrt{1-x^2} + \frac{1}{2}(\arcsin x)^2 + C$

73. $\frac{1}{8} \ln|x-2| - \frac{1}{16} \ln(x^2+4) - \frac{1}{8} \tan^{-1}(x/2) + C$

75. $2(x-2)\sqrt{1+e^x} + 2 \ln \frac{\sqrt{1+e^x}+1}{\sqrt{1+e^x}-1} + C$

77. $\frac{2}{3} \tan^{-1}(x^{3/2}) + C$

79. $\frac{1}{3}x \sin^3 x + \frac{1}{3} \cos x - \frac{1}{9} \cos^3 x + C$

81. $xe^{x^2} + C$

EXERCISES 8.6 ■ PAGE 529

1. $(-1/x)\sqrt{7-2x^2} - \sqrt{2} \sin^{-1}(\sqrt{2}x/\sqrt{7}) + C$

3. $\frac{1}{2\pi} \sec(\pi x) \tan(\pi x) + \frac{1}{2\pi} \ln|\sec(\pi x) + \tan(\pi x)| + C$

5. $\pi/4$

7. $\frac{1}{2\pi} \tan^2(\pi x) + \frac{1}{\pi} \ln|\cos(\pi x)| + C$

9. $-\sqrt{4x^2+9}/(9x) + C$

11. $e-2$

13. $-\frac{1}{2} \tan^2(1/z) - \ln|\cos(1/z)| + C$

15. $\frac{1}{2}(e^{2x}+1) \arctan(e^x) - \frac{1}{2}e^x + C$

17. $\frac{2y-1}{8} \sqrt{6+4y-4y^2} + \frac{7}{8} \sin^{-1}\left(\frac{2y-1}{\sqrt{7}}\right)$

$-\frac{1}{12}(6+4y-4y^2)^{3/2} + C$

19. $\frac{1}{9} \sin^3 x [3 \ln(\sin x) - 1] + C$

21. $\frac{1}{2\sqrt{3}} \ln \left| \frac{e^x + \sqrt{3}}{e^x - \sqrt{3}} \right| + C$

23. $\frac{1}{4} \tan x \sec^3 x + \frac{3}{8} \tan x \sec x + \frac{3}{8} \ln |\sec x + \tan x| + C$

25. $\frac{1}{2} (\ln x) \sqrt{4 + (\ln x)^2} + 2 \ln [\ln x + \sqrt{4 + (\ln x)^2}] + C$

27. $\sqrt{e^{2x} - 1} - \cos^{-1}(e^{-x}) + C$

29. $\frac{1}{5} \ln |x^5 + \sqrt{x^{10} - 2}| + C \quad 31. 2\pi^2$

35. $\frac{1}{3} \tan x \sec^2 x + \frac{2}{3} \tan x + C$

37. $\frac{1}{4}x(x^2 + 2)\sqrt{x^2 + 4} - 2 \ln(\sqrt{x^2 + 4} + x) + C$

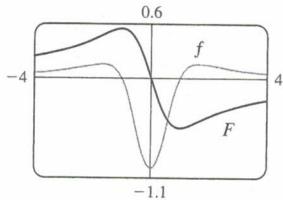
39. $\frac{1}{10}(1 + 2x)^{5/2} - \frac{1}{6}(1 + 2x)^{3/2} + C$

41. $-\ln |\cos x| - \frac{1}{2} \tan^2 x + \frac{1}{4} \tan^4 x + C$

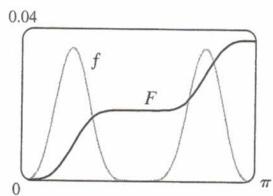
43. (a) $-\ln \left| \frac{1 + \sqrt{1 - x^2}}{x} \right| + C$;

both have domain $(-1, 0) \cup (0, 1)$

45. $F(x) = \frac{1}{2} \ln(x^2 - x + 1) - \frac{1}{2} \ln(x^2 + x + 1);$

max. at -1 , min. at 1 ; IP at $-1.7, 0$, and 1.7 

47. $F(x) = -\frac{1}{10} \sin^3 x \cos^7 x - \frac{3}{80} \sin x \cos^7 x + \frac{1}{160} \sin x \cos^5 x + \frac{1}{128} \sin x \cos^3 x + \frac{3}{256} \sin x \cos x + \frac{3}{256} x;$
max. at π , min. at 0 ; IP at $0.7, \pi/2$, and 2.5



EXERCISES 8.7 ■ PAGE 541

1. (a) $L_2 = 6, R_2 = 12, M_2 \approx 9.6$

(b) L_2 is an underestimate, R_2 and M_2 are overestimates.

(c) $T_2 = 9 < I$ (d) $L_n < T_n < I < M_n < R_n$

3. (a) $T_4 \approx 0.895759$ (underestimate)

(b) $M_4 \approx 0.908907$ (overestimate)

$T_4 < I < M_4$

5. (a) $5.932957, E_M \approx -0.063353$

(b) $5.869247, E_S \approx 0.000357$

7. (a) 2.413790 (b) 2.411453 (c) 2.412232

9. (a) 0.146879 (b) 0.147391 (c) 0.147219

11. (a) 0.451948 (b) 0.451991 (c) 0.451976

13. (a) 4.513618 (b) 4.748256 (c) 4.675111

15. (a) -0.495333 (b) -0.543321 (c) -0.526123

17. (a) 1.064275 (b) 1.067416 (c) 1.074915

19. (a) $T_8 \approx 0.902333, M_8 \approx 0.905620$

(b) $|E_T| \leq 0.0078, |E_M| \leq 0.0039$

(c) $n = 71$ for $T_n, n = 50$ for M_n

21. (a) $T_{10} \approx 1.983524, E_T \approx 0.016476$

$M_{10} \approx 2.008248, E_M \approx -0.008248$

$S_{10} \approx 2.000110, E_S \approx -0.000110$

(b) $|E_T| \leq 0.025839, |E_M| \leq 0.012919, |E_S| \leq 0.000170$

(c) $n = 509$ for $T_n, n = 360$ for $M_n, n = 22$ for S_n

23. (a) 2.8 (b) 7.954926518 (c) 0.2894

(d) 7.954926521 (e) The actual error is much smaller.

(f) 10.9 (g) 7.953789422 (h) 0.0593

(i) The actual error is smaller. (j) $n \geq 50$

25.

n	L_n	R_n	T_n	M_n
5	0.742943	1.286599	1.014771	0.992621
10	0.867782	1.139610	1.003696	0.998152
20	0.932967	1.068881	1.000924	0.999538

n	E_L	E_R	E_T	E_M
5	0.257057	-0.286599	-0.014771	0.007379
10	0.132218	-0.139610	-0.003696	0.001848
20	0.067033	-0.068881	-0.000924	0.000462

Observations are the same as after Example 1.

27.

n	T_n	M_n	S_n
6	6.695473	6.252572	6.403292
12	6.474023	6.363008	6.400206

n	E_T	E_M	E_S
6	-0.295473	0.147428	-0.003292
12	-0.074023	0.036992	-0.000206

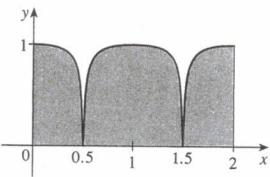
Observations are the same as after Example 1.

29. (a) 19.8 (b) 20.6 (c) 20.53

31. (a) 23.44 (b) 0.3413 **33.** 37.73 ft/s

35. 10,177 megawatt-hours **37.** 828 **39.** 6.0 **41.** 59.4

43.



EXERCISES 8.8 ■ PAGE 551

Abbreviations: C, convergent; D, divergent

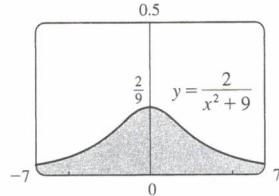
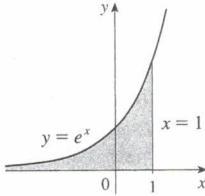
1. (a) Infinite interval (b) Infinite discontinuity

(c) Infinite discontinuity (d) Infinite interval

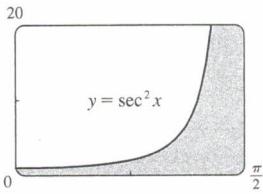
3. $\frac{1}{2} - \frac{1}{(2t^2)}$; 0.495, 0.49995, 0.499995; 0.5

5. $\frac{1}{12}$ 7. D 9. $2e^{-2}$ 11. D 13. 0 15. D
 17. D 19. $\frac{1}{25}$ 21. D 23. $\pi/9$
 25. $\frac{1}{2}$ 27. D 29. $\frac{32}{3}$ 31. D 33. $\frac{75}{4}$
 35. D 37. $-2/e$ 39. $\frac{8}{3} \ln 2 - \frac{8}{9}$

41. e 43. $2\pi/3$

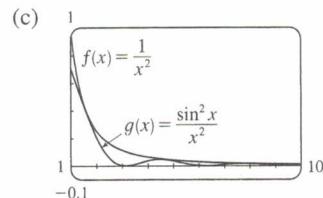


45. Infinite area

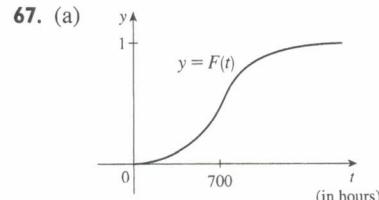


t	$\int_1^t [(\sin^2 x)/x^2] dx$
2	0.447453
5	0.577101
10	0.621306
100	0.668479
1,000	0.672957
10,000	0.673407

It appears that the integral is convergent.



49. C 51. D 53. D 55. π 57. $p < 1, 1/(1-p)$
 59. $p > -1, -1/(p+1)^2$ 65. $\sqrt{2GM/R}$



- (b) The rate at which the fraction $F(t)$ increases as t increases
 (c) 1; all bulbs burn out eventually
 69. 1000
 71. (a) $F(s) = 1/s, s > 0$ (b) $F(s) = 1/(s-1), s > 1$
 (c) $F(s) = 1/s^2, s > 0$
 77. $C = 1; \ln 2$ 79. No

CHAPTER 8 REVIEW ■ PAGE 554

True-False Quiz

1. False 3. False 5. False 7. False
 9. (a) True (b) False 11. False 13. False

Exercises

1. $5 + 10 \ln \frac{2}{3}$ 3. $\ln 2$ 5. $\frac{2}{15}$
 7. $-\cos(\ln t) + C$ 9. $\frac{64}{5} \ln 4 - \frac{124}{25}$
 11. $\sqrt{3} - \frac{1}{3}\pi$ 13. $3e^{\sqrt[3]{x}} (\sqrt[3]{x^2} - 2\sqrt[3]{x} + 2) + C$
 15. $-\frac{1}{2} \ln|x| + \frac{3}{2} \ln|x+2| + C$
 17. $x \sec x - \ln|\sec x + \tan x| + C$
 19. $\frac{1}{18} \ln(9x^2 + 6x + 5) + \frac{1}{9} \tan^{-1}[\frac{1}{2}(3x+1)] + C$
 21. $\ln|x-2 + \sqrt{x^2-4x}| + C$
 23. $\ln \left| \frac{\sqrt{x^2+1}-1}{x} \right| + C$
 25. $\frac{3}{2} \ln(x^2+1) - 3 \tan^{-1}x + \sqrt{2} \tan^{-1}(x/\sqrt{2}) + C$
 27. $\frac{2}{5}$ 29. 0 31. $6 - \frac{3}{2}\pi$
 33. $\frac{x}{\sqrt{4-x^2}} - \sin^{-1}\left(\frac{x}{2}\right) + C$
 35. $4\sqrt{1+\sqrt{x}} + C$ 37. $\frac{1}{2} \sin 2x - \frac{1}{8} \cos 4x + C$
 39. $\frac{1}{8}e - \frac{1}{4}$ 41. $\frac{1}{36}$ 43. D
 45. $4 \ln 4 - 8$ 47. $-\frac{4}{3}$ 49. $\pi/4$
 51. $(x+1) \ln(x^2+2x+2) + 2 \arctan(x+1) - 2x + C$
 53. 0
 55. $\frac{1}{4}(2x-1)\sqrt{4x^2-4x-3} - \ln|2x-1+\sqrt{4x^2-4x-3}| + C$
 57. $\frac{1}{2} \sin x \sqrt{4+\sin^2 x} + 2 \ln(\sin x + \sqrt{4+\sin^2 x}) + C$
 61. No
 63. (a) 1.925444 (b) 1.920915 (c) 1.922470
 65. (a) 0.01348, $n \geq 368$ (b) 0.00674, $n \geq 260$
 67. 8.6 mi
 69. (a) 3.8 (b) 1.7867, 0.000646 (c) $n \geq 30$
 71. C 73. 2 75. $\frac{3}{16}\pi^2$

PROBLEMS PLUS ■ PAGE 558

1. About 1.85 inches from the center 3. 0
 7. $f(\pi) = -\pi/2$ 11. $(b^b a^{-a})^{1/(b-a)} e^{-1}$
 13. $2 - \sin^{-1}(2/\sqrt{5})$

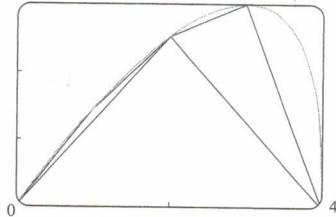
CHAPTER 9

EXERCISES 9.1 ■ PAGE 566

1. $4\sqrt{5}$ 3. $\int_0^{2\pi} \sqrt{1+\sin^2 x} dx$ 5. $\int_1^4 \sqrt{9y^4 + 6y^2 + 2} dy$
 7. $\frac{2}{243}(82\sqrt{82} - 1)$ 9. $\frac{1261}{240}$ 11. $\frac{32}{3}$
 13. $\ln(\sqrt{2}+1)$ 15. $\ln 3 - \frac{1}{2}$
 17. $\sqrt{1+e^2} - \sqrt{2} + \ln(\sqrt{1+e^2} - 1) - 1 - \ln(\sqrt{2}-1)$
 19. $\sqrt{2} + \ln(1+\sqrt{2})$ 21. $\frac{46}{3}$ 23. 5.115840

25. 1.569619

27. (a), (b) 3

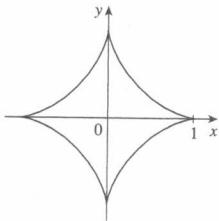


$$\begin{aligned}L_1 &= 4, \\L_2 &\approx 6.43, \\L_4 &\approx 7.50\end{aligned}$$

(c) $\int_0^4 \sqrt{1 + [4(3-x)/(3(4-x)^{2/3})]^2} dx$ (d) 7.7988

29. $\sqrt{5} - \ln(\frac{1}{2}(1 + \sqrt{5})) - \sqrt{2} + \ln(1 + \sqrt{2})$

31. 6



33. $s(x) = \frac{2}{27}[(1+9x)^{3/2} - 10\sqrt{10}]$ 35. $2\sqrt{2}(\sqrt{1+x} - 1)$

37. 209.1 m 39. 29.36 in. 41. 12.4

EXERCISES 9.2 ■ PAGE 573

1. (a) $\int_0^1 2\pi x^4 \sqrt{1+16x^6} dx$ (b) $\int_0^1 2\pi x \sqrt{1+16x^6} dx$

3. (a) $\int_0^1 2\pi \tan^{-1} x \sqrt{1+\frac{1}{(1+x^2)^2}} dx$

(b) $\int_0^1 2\pi x \sqrt{1+\frac{1}{(1+x^2)^2}} dx$

5. $\frac{1}{27}\pi(145\sqrt{145}-1)$ 7. $\frac{98}{3}\pi$

9. $2\sqrt{1+\pi^2} + (2/\pi) \ln(\pi + \sqrt{1+\pi^2})$ 11. $\frac{21}{2}\pi$

13. $\frac{1}{27}\pi(145\sqrt{145}-10\sqrt{10})$ 15. πa^2

17. 9.023754 19. 13.527296

21. $\frac{1}{4}\pi[4\ln(\sqrt{17}+4)-4\ln(\sqrt{2}+1)-\sqrt{17}+4\sqrt{2}]$

23. $\frac{1}{6}\pi[\ln(\sqrt{10}+3)+3\sqrt{10}]$

27. (a) $\frac{1}{3}\pi a^2$ (b) $\frac{56}{45}\pi\sqrt{3}a^2$

29. (a) $2\pi \left[b^2 + \frac{a^2 b \sin^{-1}(\sqrt{a^2 - b^2}/a)}{\sqrt{a^2 - b^2}} \right]$

(b) $2\pi \left[a^2 + \frac{ab^2 \sin^{-1}(\sqrt{b^2 - a^2}/b)}{\sqrt{b^2 - a^2}} \right]$

31. $\int_a^b 2\pi[c-f(x)]\sqrt{1+[f'(x)]^2} dx$ 33. $4\pi^2 r^2$

EXERCISES 9.3 ■ PAGE 583

1. (a) 187.5 lb/ft² (b) 1875 lb (c) 562.5 lb

3. 6000 lb 5. 6.7×10^4 N 7. 9.8×10^3 N

9. 1.2×10^4 lb 11. $\frac{2}{3}\delta ah$ 13. 5.27×10^5 N

15. (a) 314 N (b) 353 N

17. (a) 5.63×10^3 lb (b) 5.06×10^4 lb

(c) 4.88×10^4 lb (d) 3.03×10^5 lb

19. 2.5×10^5 N 21. $230; \frac{23}{7}$ 23. $10; 1; (\frac{1}{21}, \frac{10}{21})$

25. $(0, 1.6)$ 27. $\left(\frac{1}{e-1}, \frac{e+1}{4}\right)$ 29. $(\frac{9}{20}, \frac{9}{20})$

31. $\left(\frac{\pi\sqrt{2}-4}{4(\sqrt{2}-1)}, \frac{1}{4(\sqrt{2}-1)}\right)$ 33. $(2, 0)$

35. $60; 160; (\frac{8}{3}, 1)$ 37. $(0.781, 1.330)$ 41. $(0, \frac{1}{12})$

45. $\frac{1}{3}\pi r^2 h$

EXERCISES 9.4 ■ PAGE 589

1. \$38,000 3. \$43,866,933.33 5. \$407.25

7. \$12,000 9. 3727; \$37,753

11. $\frac{2}{3}(16\sqrt{2}-8) \approx \9.75 million 13. $\frac{(1-k)(b^{2-k}-a^{2-k})}{(2-k)(b^{1-k}-a^{1-k})}$

15. 1.19×10^{-4} cm³/s

17. 6.60 L/min 19. 5.77 L/min

EXERCISES 9.5 ■ PAGE 596

1. (a) The probability that a randomly chosen tire will have a lifetime between 30,000 and 40,000 miles

(b) The probability that a randomly chosen tire will have a lifetime of at least 25,000 miles

3. (a) $f(x) \geq 0$ for all x and $\int_{-\infty}^{\infty} f(x) dx = 1$

(b) $1 - \frac{3}{8}\sqrt{3} \approx 0.35$

5. (a) $1/\pi$ (b) $\frac{1}{2}$

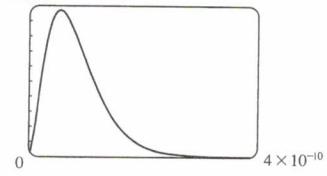
7. (a) $f(x) \geq 0$ for all x and $\int_{-\infty}^{\infty} f(x) dx = 1$ (b) 5

11. (a) $e^{-4/2.5} \approx 0.20$ (b) $1 - e^{-2/2.5} \approx 0.55$ (c) If you aren't served within 10 minutes, you get a free hamburger.13. $\approx 44\%$

15. (a) 0.0668 (b) $\approx 5.21\%$

17. ≈ 0.9545

19. (b) 0; a_0 (c) 1×10^{10}



(d) $1 - 41e^{-8} \approx 0.986$ (e) $\frac{3}{2}a_0$

CHAPTER 9 REVIEW ■ PAGE 598

Exercises

1. $\frac{15}{2}$ 3. (a) $\frac{21}{16}$ (b) $\frac{41}{10}\pi$ 5. 3.292287 7. $\frac{124}{5}$

9. ≈ 458 lb 11. $(\frac{8}{5}, 1)$ 13. $(2, \frac{2}{3})$ 15. $2\pi^2$

17. \$7166.67

19. (a) $f(x) \geq 0$ for all x and $\int_{-\infty}^{\infty} f(x) dx = 1$ (b) ≈ 0.3455 (c) 5, yes

21. (a) $1 - e^{-3/8} \approx 0.31$ (b) $e^{-5/4} \approx 0.29$ (c) $8 \ln 2 \approx 5.55$ min

PROBLEMS PLUS ■ PAGE 600

1. $\frac{2}{3}\pi - \frac{1}{2}\sqrt{3}$

3. (a) $2\pi r(r \pm d)$ (b) $\approx 3.36 \times 10^6 \text{ mi}^2$

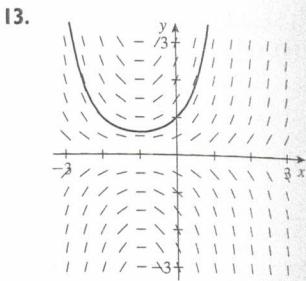
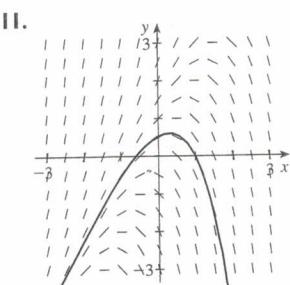
(d) $\approx 7.84 \times 10^7 \text{ mi}^2$

5. (a) $P(z) = P_0 + g \int_0^z \rho(x) dx$

(b) $(P_0 - \rho_0 g H)(\pi r^2) + \rho_0 g H e^{L/H} \int_r^r e^{x/H} \cdot 2\sqrt{r^2 - x^2} dx$

7. Height $\sqrt{2} b$, volume $(\frac{28}{27}\sqrt{6} - 2)\pi b^3$ 9. 0.14 m

11. $2/\pi, 1/\pi$



CHAPTER 10

EXERCISES 10.1 ■ PAGE 607

3. (a) $\frac{1}{2}, -1$ 5. (d)

7. (a) It must be either 0 or decreasing

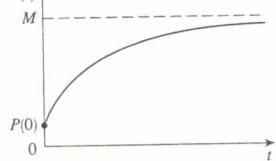
(c) $y = 0$ (d) $y = 1/(x+2)$

9. (a) $0 < P < 4200$ (b) $P > 4200$

(c) $P = 0, P = 4200$

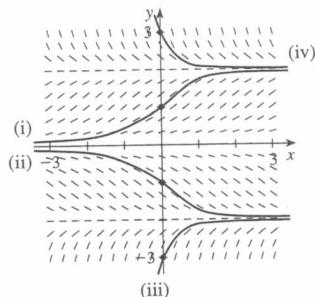
13. (a) At the beginning; stays positive, but decreases

(c)



EXERCISES 10.2 ■ PAGE 614

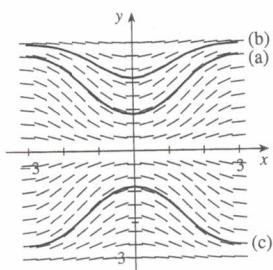
1. (a)



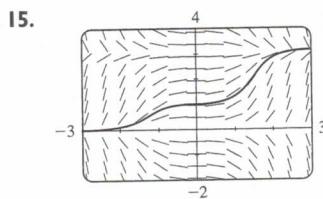
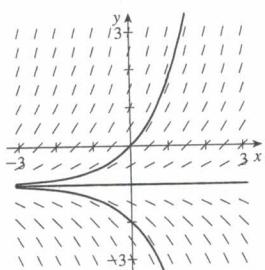
(b) $y = 0$,
 $y = 2$,
 $y = -2$

3. III 5. IV

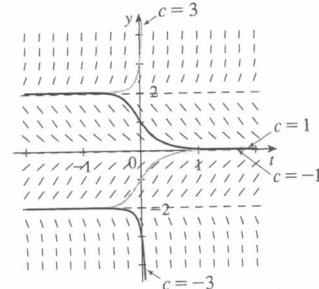
7.



9.

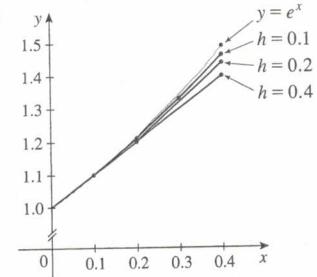


15. $-2 \leq c \leq 2; -2, 0, 2$



19. (a) (i) 1.4 (ii) 1.44 (iii) 1.4641

(b) Underestimates



(c) (i) 0.0918 (ii) 0.0518 (iii) 0.0277

It appears that the error is also halved (approximately).

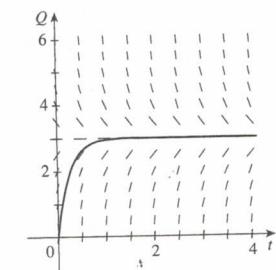
21. $-1, -3, -6.5, -12.25$ 23. 1.7616

25. (a) (i) 3 (ii) 2.3928 (iii) 2.3701 (iv) 2.3681

(c) (i) -0.6321 (ii) -0.0249 (iii) -0.0022 (iv) -0.0002

It appears that the error is also divided by 10 (approximately).

27. (a), (d)



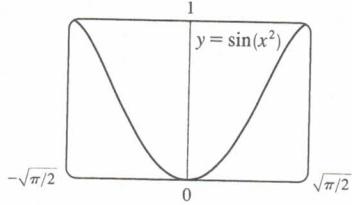
(b) 3

(c) Yes; $Q = 3$

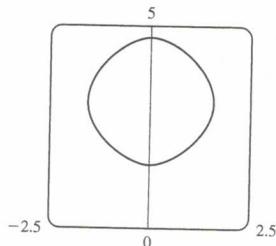
(e) 2.77 C

EXERCISES 10.3 ■ PAGE 622

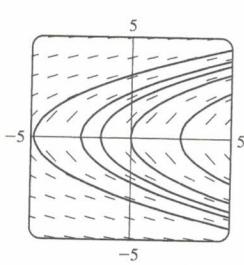
1. $y = Kx$
3. $y = K\sqrt{x^2 + 1}$
5. $y + \ln |\sec y| = \frac{1}{3}x^3 + x + C$
7. $y = \pm\sqrt{[3(te^t - e^t + C)]^{2/3} - 1}$
9. $u = Ae^{2t+t^2/2} - 1$
11. $y = -\sqrt{x^2 + 9}$
13. $\cos x + x \sin x = y^2 + \frac{1}{3}e^{3y} + \frac{2}{3}$
15. $u = -\sqrt{t^2 + \tan t + 25}$
17. $y = \frac{4a}{\sqrt{3}} \sin x - a$
19. $y = e^{x^2/2}$
21. $y = Ke^x - x - 1$
23. (a) $\sin^{-1}y = x^2 + C$
- (b) $y = \sin(x^2)$, $-\sqrt{\pi/2} \leq x \leq \sqrt{\pi/2}$
- (c) No



25. $\cos y = \cos x - 1$

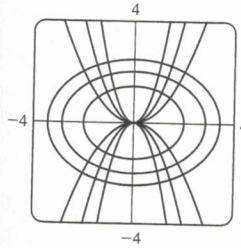


27. (a), (c)

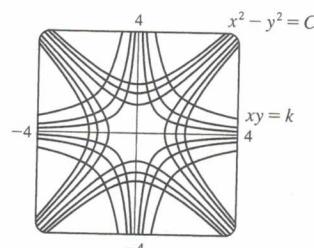


(b) $y = \pm\sqrt{2(x + C)}$

29. $y = Cx^2$



31. $x^2 - y^2 = C$



33. $Q(t) = 3 - 3e^{-4t}$; 3 35. $P(t) = M - Me^{-kt}$; M

37. (a) $x = a - \frac{4}{(kt + 2/\sqrt{a})^2}$

(b) $t = \frac{2}{k\sqrt{a-b}} \left(\tan^{-1} \sqrt{\frac{b}{a-b}} - \tan^{-1} \sqrt{\frac{b-x}{a-b}} \right)$

39. (a) $C(t) = (C_0 - r/k)e^{-kt} + r/k$

(b) r/k ; the concentration approaches r/k regardless of the value of C_0

41. (a) $15e^{-t/100}$ kg (b) $15e^{-0.2} \approx 12.3$ kg

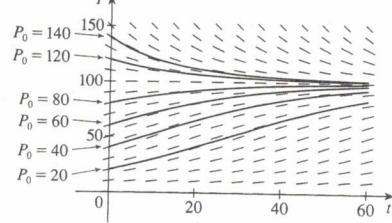
EXERCISES 10.4 ■ PAGE 634

43. About 4.9% 45. g/k

47. (a) $dA/dt = k\sqrt{A}(M - A)$ (b) $A(t) = M \left(\frac{Ce^{\sqrt{M}kt} - 1}{Ce^{\sqrt{M}kt} + 1} \right)^2$,
where $C = \frac{\sqrt{M} + \sqrt{A_0}}{\sqrt{M} - \sqrt{A_0}}$ and $A_0 = A(0)$

EXERCISES 10.4 ■ PAGE 634

1. (a) 100; 0.05 (b) Where P is close to 0 or 100; on the line $P = 50$; $0 < P_0 < 100$; $P_0 > 100$
- (c)



Solutions approach 100; some increase and some decrease, some have an inflection point but others don't; solutions with $P_0 = 20$ and $P_0 = 40$ have inflection points at $P = 50$

(d) $P = 0$, $P = 100$; other solutions move away from $P = 0$ and toward $P = 100$

3. (a) 3.23×10^7 kg (b) ≈ 1.55 years

5. (a) $dP/dt = \frac{1}{265}P(1 - P/100)$, P in billions

(b) 5.49 billion (c) In billions: 7.81, 27.72

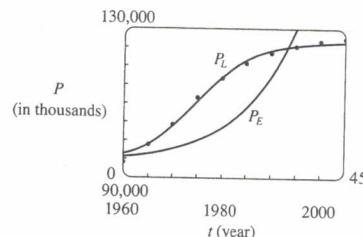
(d) In billions: 5.48, 7.61, 22.41

7. (a) $dy/dt = ky(1 - y)$ (b) $y = \frac{y_0}{y_0 + (1 - y_0)e^{-kt}}$

(c) 3:36 PM

11. $P_E(t) = 1578.3(1.0933)^t + 94,000$;

$$P_L(t) = \frac{32,658.5}{1 + 12.75e^{-0.1706t}} + 94,000$$



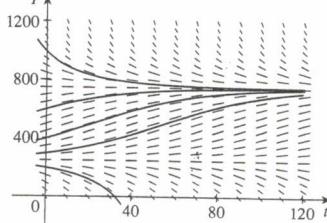
13. (a) $P(t) = \frac{m}{k} + \left(P_0 - \frac{m}{k} \right) e^{kt}$ (b) $m < kP_0$

(c) $m = kP_0$, $m > kP_0$ (d) Declining

15. (a) Fish are caught at a rate of 15 per week.

(b) See part (d) (c) $P = 250$, $P = 750$

(d)

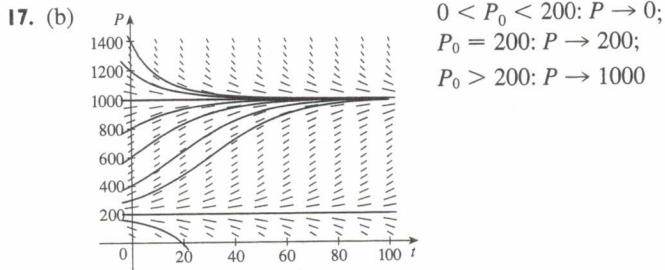
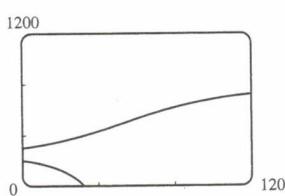


$0 < P_0 < 250$: $P \rightarrow 0$;

$P_0 = 250$: $P \rightarrow 250$;

$P_0 > 250$: $P \rightarrow 750$

(e) $P(t) = \frac{250 - 750ke^{t/25}}{1 - ke^{t/25}}$
where $k = \frac{1}{11}, -\frac{1}{9}$



(c) $P(t) = \frac{m(K - P_0) + K(P_0 - m)e^{(K-m)(k/K)t}}{K - P_0 + (P_0 - m)e^{(K-m)(k/K)t}}$

19. (a) $P(t) = P_0 e^{(k/r)[\sin(rt - \phi) + \sin \phi]}$ (b) Does not exist

EXERCISES 10.5 ■ PAGE 642

1. Yes 3. No 5. $y = \frac{2}{3}e^x + Ce^{-2x}$

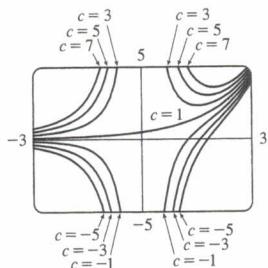
7. $y = x^2 \ln|x| + Cx^2$ 9. $y = \frac{2}{3}\sqrt{x} + C/x$

11. $y = \frac{\int \sin(x^2) dx + C}{\sin x}$ 13. $u = \frac{t^2 + 2t + 2C}{2(t+1)}$

15. $y = -x - 1 + 3e^x$

19. $y = -x \cos x - x$

21. $y = \frac{(x-1)e^x + C}{x^2}$

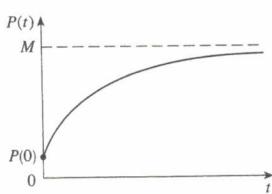


25. $y = \pm \left(Cx^4 + \frac{2}{5x} \right)^{-1/2}$

27. (a) $I(t) = 4 - 4e^{-5t}$ (b) $4 - 4e^{-1/2} \approx 1.57 \text{ A}$

29. $Q(t) = 3(1 - e^{-4t})$, $I(t) = 12e^{-4t}$

31. $P(t) = M + Ce^{-kt}$



33. $y = \frac{2}{5}(100 + 2t) - 40,000(100 + 2t)^{-3/2}; 0.2275 \text{ kg/L}$

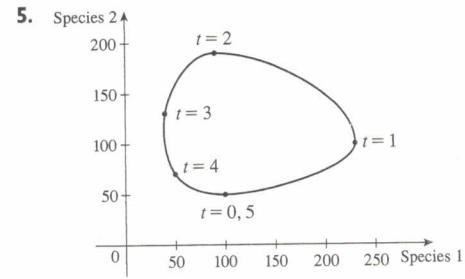
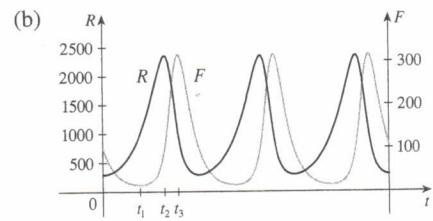
35. (b) mg/c (c) $(mg/c)[t + (m/c)e^{-ct/m}] - m^2g/c^2$

EXERCISES 10.6 ■ PAGE 648

1. (a) x = predators, y = prey; growth is restricted only by predators, which feed only on prey.

(b) x = prey, y = predators; growth is restricted by carrying capacity and by predators, which feed only on prey.

3. (a) The rabbit population starts at about 300, increases to 2400, then decreases back to 300. The fox population starts at 100, decreases to about 20, increases to about 315, decreases to 100, and the cycle starts again.



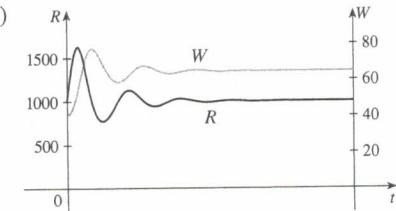
9. (a) Population stabilizes at 5000.

(b) (i) $W = 0, R = 0$: Zero populations

(ii) $W = 0, R = 5000$: In the absence of wolves, the rabbit population is always 5000.

(iii) $W = 64, R = 1000$: Both populations are stable.

(c) The populations stabilize at 1000 rabbits and 64 wolves.



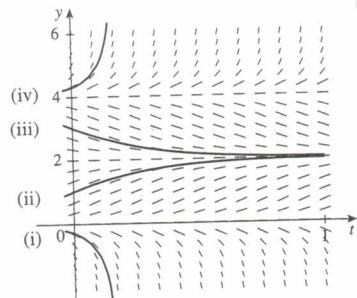
CHAPTER 10 REVIEW ■ PAGE 651

True-False Quiz

1. True 3. False 5. True 7. True

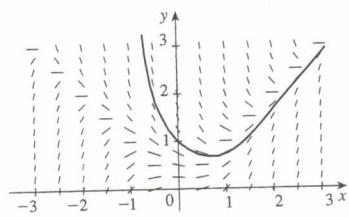
Exercises

1. (a)



(b) $0 \leq c \leq 4$
 $y = 0, y = 2, y = 4$

3. (a)



$y(0.3) \approx 0.8$

(b) 0.75676

(c) $y = x$ and $y = -x$; there is a local maximum or minimum

5. $y = (\frac{1}{2}x^2 + C)e^{-\sin x}$ 7. $y = \pm\sqrt{\ln(x^2 + 2x^{3/2} + C)}$

9. $r(t) = 5e^{t-t^2}$ 11. $y = \frac{1}{2}x(\ln x)^2 + 2x$ 13. $x = C - \frac{1}{2}y^2$

15. (a) $P(t) = \frac{2000}{1 + 19e^{-0.1t}}$; ≈ 560 (b) $t = -10 \ln \frac{2}{57} \approx 33.5$

17. (a) $L(t) = L_\infty - [L_\infty - L(0)]e^{-kt}$ (b) $L(t) = 53 - 43e^{-0.2t}$

19. 15 days 21. $k \ln h + h = (-R/V)t + C$

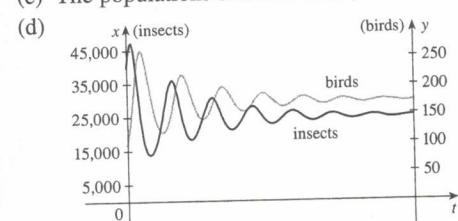
23. (a) Stabilizes at 200,000

(b) (i) $x = 0, y = 0$: Zero populations

(ii) $x = 200,000, y = 0$: In the absence of birds, the insect population is always 200,000.

(iii) $x = 25,000, y = 175$: Both populations are stable.

(c) The populations stabilize at 25,000 insects and 175 birds.



25. (a) $y = (1/k) \cosh kx + a - 1/k$ or
 $y = (1/k) \cosh kx - (1/k) \cosh kb + h$ (b) $(2/k) \sinh kb$

PROBLEMS PLUS = PAGE 654

1. $f(x) = \pm 10e^x$ 5. $y = x^{1/n}$ 7. 20°C

9. (b) $f(x) = \frac{x^2 - L^2}{4L} - \frac{1}{2}L \ln\left(\frac{x}{L}\right)$ (c) No

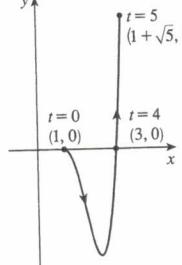
11. (a) 9.8 h (b) $31,900\pi \approx 100,000 \text{ ft}^2$; $6283 \text{ ft}^2/\text{h}$

(c) 5.1 h

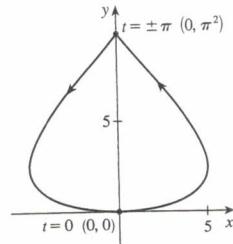
13. $x^2 + (y - 6)^2 = 25$

CHAPTER II**EXERCISES 11.1 = PAGE 662**

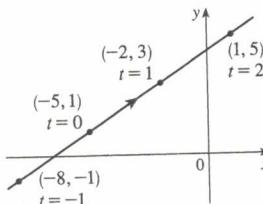
1.



3.

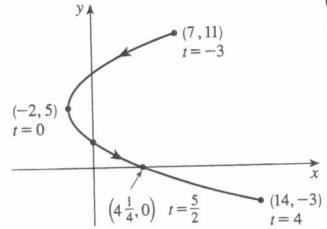


5. (a)



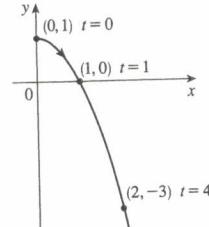
(b) $y = \frac{2}{3}x + \frac{13}{3}$

7. (a)



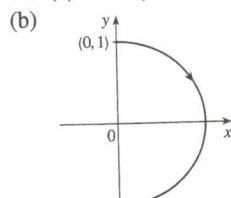
(b) $x = \frac{1}{4}(y - 5)^2 - 2$,
 $-3 \leq y \leq 11$

9. (a)

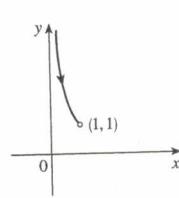


(b) $y = 1 - x^2, x \geq 0$

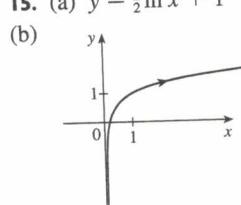
11. (a) $x^2 + y^2 = 1, x \geq 0$



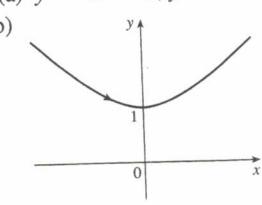
13. (a) $y = 1/x, y > 1$



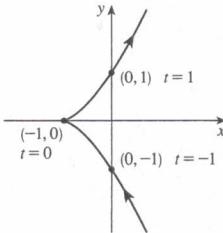
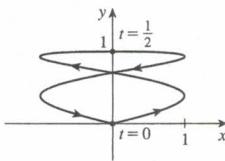
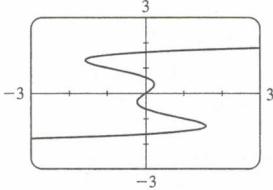
15. (a) $y = \frac{1}{2}\ln x + 1$



17. (a) $y^2 - x^2 = 1, y \geq 1$

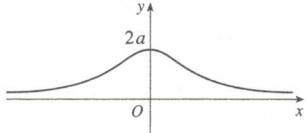


- 19.** Moves counterclockwise along the circle
 $(x - 3)^2 + (y - 1)^2 = 4$ from $(3, 3)$ to $(3, -1)$
- 21.** Moves 3 times clockwise around the ellipse
 $(x^2/25) + (y^2/4) = 1$, starting and ending at $(0, -2)$
- 23.** It is contained in the rectangle described by $1 \leq x \leq 4$ and $2 \leq y \leq 3$.

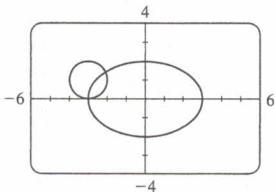
25.**27.****29.**

- 31.** (b) $x = -2 + 5t$, $y = 7 - 8t$, $0 \leq t \leq 1$
- 33.** (a) $x = 2 \cos t$, $y = 1 - 2 \sin t$, $0 \leq t \leq 2\pi$
(b) $x = 2 \cos t$, $y = 1 + 2 \sin t$, $0 \leq t \leq 6\pi$
(c) $x = 2 \cos t$, $y = 1 + 2 \sin t$, $\pi/2 \leq t \leq 3\pi/2$
- 37.** The curve $y = x^{2/3}$ is generated in (a). In (b), only the portion with $x \geq 0$ is generated, and in (c) we get only the portion with $x > 0$.

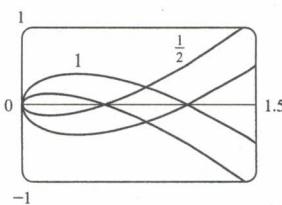
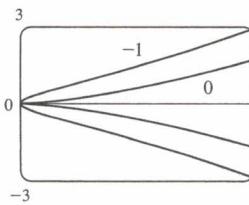
- 41.** $x = a \cos \theta$, $y = b \sin \theta$; $(x^2/a^2) + (y^2/b^2) = 1$, ellipse

43.

- 45.** (a) Two points of intersection



- (b) One collision point at $(-3, 0)$ when $t = 3\pi/2$
(c) There are still two intersection points, but no collision point.
47. For $c = 0$, there is a cusp; for $c > 0$, there is a loop whose size increases as c increases.

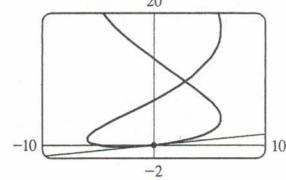


- 49.** As n increases, the number of oscillations increases; a and b determine the width and height.

EXERCISES 11.2 ■ PAGE 672

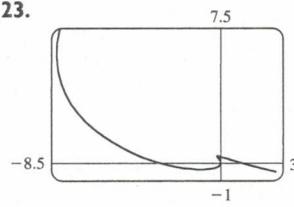
1. $\frac{2t+1}{t \cos t + \sin t}$ **3.** $y = -x$

5. $y = -(2/e)x + 3$ **7.** $y = 2x + 1$
9. $y = \frac{1}{6}x$

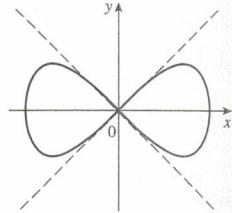


- 11.** $1 + \frac{3}{2}t$, $3/(4t)$, $t > 0$
13. $-e^{-t}$, $e^{-t}/(1 - e^t)$, $t < 0$
15. $-\frac{3}{2} \tan t$, $-\frac{3}{4} \sec^3 t$, $\pi/2 < t < 3\pi/2$
17. Horizontal at $(6, \pm 16)$, vertical at $(10, 0)$
19. Horizontal at $(\pm\sqrt{2}, \pm 1)$ (four points), vertical at $(\pm 2, 0)$

21. $(0.6, 2)$; $(5 \cdot 6^{-6/5}, e^{6^{-1/5}})$

23.

25. $y = x$, $y = -x$



27. (a) $d \sin \theta / (r - d \cos \theta)$ **29.** $(\frac{16}{27}, \frac{29}{9})$, $(-2, -4)$

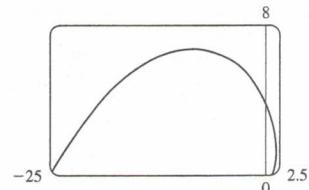
31. πab **33.** $3 - e$ **35.** $2\pi r^2 + \pi d^2$

37. $\int_1^2 \sqrt{1 + 4t^2} dt \approx 3.1678$

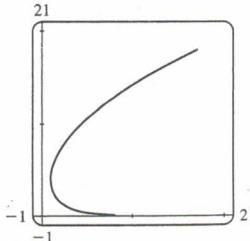
39. $\int_0^{2\pi} \sqrt{3 - 2 \sin t - 2 \cos t} dt \approx 10.0367$ **41.** $4\sqrt{2} - 2$

43. $-\sqrt{10}/3 + \ln(3 + \sqrt{10}) + \sqrt{2} - \ln(1 + \sqrt{2})$

45. $\sqrt{2}(e^\pi - 1)$

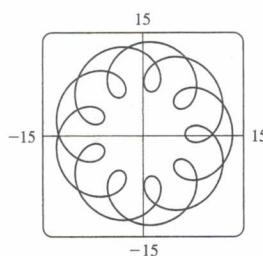


47. $e^3 + 11 - e^{-8}$



49. 612.3053 **51.** $6\sqrt{2}$, $\sqrt{2}$

55. (a)



$$t \in [0, 4\pi]$$

$$(b) \approx 294$$

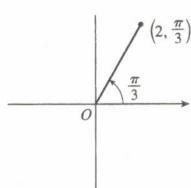
$$57. \int_0^1 2\pi(t^2 + 1)e^t \sqrt{e^{2t}(t+1)^2(t^2 + 2t + 2)} dt \approx 103.5999$$

$$59. \frac{2}{1215}\pi(247\sqrt{13} + 64) \quad 61. \frac{6}{5}\pi a^2 \quad 63. 59.101$$

$$65. \frac{24}{5}\pi(949\sqrt{26} + 1) \quad 71. \frac{1}{4}$$

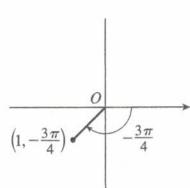
EXERCISES 11.3 ■ PAGE 683

1. (a)



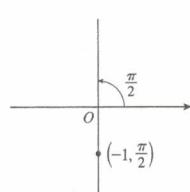
$$(2, 7\pi/3), (-2, 4\pi/3)$$

(b)



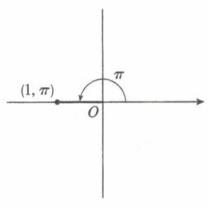
$$(1, 5\pi/4), (-1, \pi/4)$$

(c)



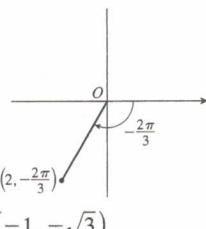
$$(1, 3\pi/2), (-1, 5\pi/2)$$

3. (a)



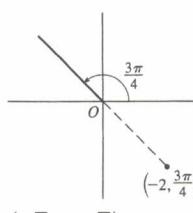
$$(-1, 0)$$

(b)



$$(-1, -\sqrt{3})$$

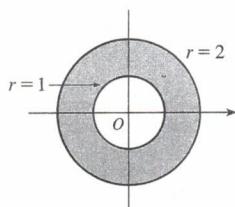
(c)



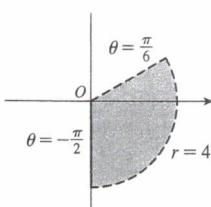
$$(\sqrt{2}, -\sqrt{2})$$

$$5. (a) (i) (2\sqrt{2}, 7\pi/4) \quad (ii) (-2\sqrt{2}, 3\pi/4) \\ (b) (i) (2, 2\pi/3) \quad (ii) (-2, 5\pi/3)$$

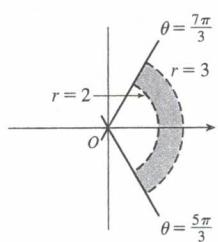
7.



9.



11.



$$13. 2\sqrt{3} \quad 15. \text{Circle, center } O, \text{ radius } 2$$

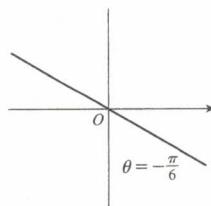
$$17. \text{Circle, center } (0, \frac{3}{2}), \text{ radius } \frac{3}{2}$$

19. Horizontal line, 1 unit above the x -axis

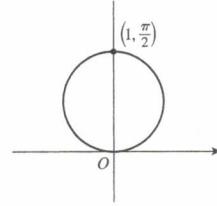
$$21. r = 3 \sec \theta \quad 23. r = -\cot \theta \csc \theta$$

$$27. (a) \theta = \pi/6 \quad (b) x = 3$$

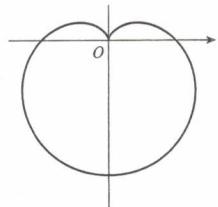
29.



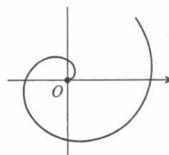
31.



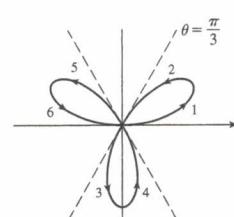
33.



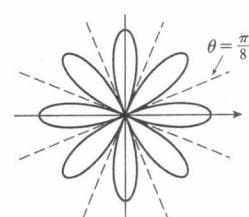
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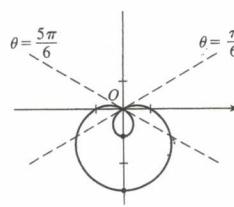
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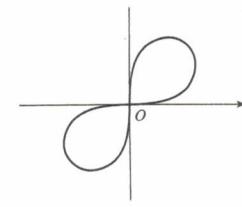
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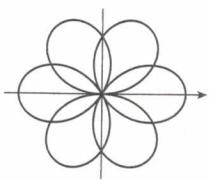
41.



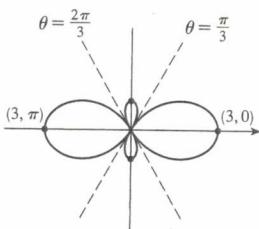
43.



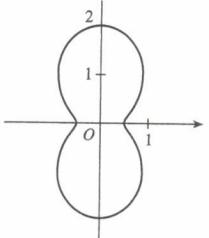
45.



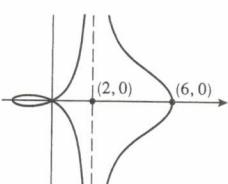
47.



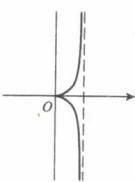
49.



51.



53.



55. (a) For $c < -1$, the inner loop begins at $\theta = \sin^{-1}(-1/c)$ and ends at $\theta = \pi - \sin^{-1}(-1/c)$; for $c > 1$, it begins at $\theta = \pi + \sin^{-1}(1/c)$ and ends at $\theta = 2\pi - \sin^{-1}(1/c)$.

57. $\sqrt{3}$ 59. $-\pi$

61. 1

63. Horizontal at $(3/\sqrt{2}, \pi/4), (-3/\sqrt{2}, 3\pi/4)$; vertical at $(3, 0), (0, \pi/2)$

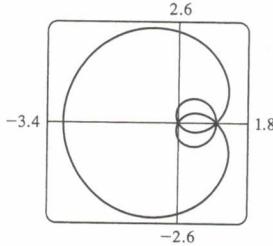
65. Horizontal at $(\frac{3}{2}, \pi/3), (0, \pi)$ [the pole], and $(\frac{3}{2}, 5\pi/3)$;

vertical at $(2, 0), (\frac{1}{2}, 2\pi/3), (\frac{1}{2}, 4\pi/3)$

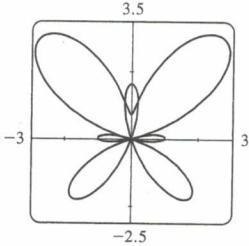
67. Horizontal at $(3, \pi/2), (1, 3\pi/2)$; vertical at $(\frac{3}{2} + \frac{1}{2}\sqrt{3}, \alpha), (\frac{3}{2} + \frac{1}{2}\sqrt{3}, \pi - \alpha)$ where $\alpha = \sin^{-1}(-\frac{1}{2} + \frac{1}{2}\sqrt{3})$

69. Center $(b/2, a/2)$, radius $\sqrt{a^2 + b^2}/2$

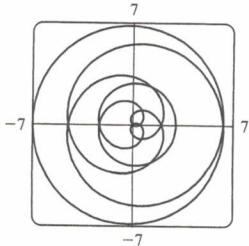
71.



73.



75.



77. By counterclockwise rotation through angle $\pi/6, \pi/3$, or α about the origin

79. (a) A rose with n loops if n is odd and $2n$ loops if n is even
 (b) Number of loops is always $2n$

81. For $0 < a < 1$, the curve is an oval, which develops a dimple as $a \rightarrow 1^-$. When $a > 1$, the curve splits into two parts, one of which has a loop.

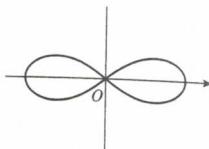
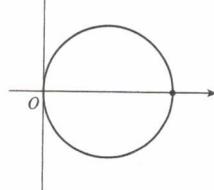
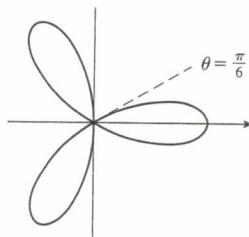
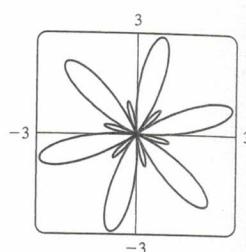
EXERCISES 11.4 ■ PAGE 689

1. $\pi^5/10,240$ 9. $\frac{9}{4}\pi$ 3. $\pi/12 + \frac{1}{8}\sqrt{3}$

11. 4

5. π^2

11. 4

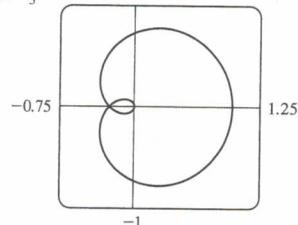
7. $\frac{41}{4}\pi$ 13. π 15. 3π 17. $\frac{1}{8}\pi$ 25. $4\sqrt{3} - \frac{4}{3}\pi$ 33. $1 - \frac{1}{2}\sqrt{2}$ 35. $\frac{1}{4}(\pi + 3\sqrt{3})$ 37. $(\frac{3}{2}, \pi/6), (\frac{3}{2}, 5\pi/6)$, and the pole

39. $(1, \theta)$ where $\theta = \pi/12, 5\pi/12, 13\pi/12, 17\pi/12$ and $(-1, \theta)$ where $\theta = 7\pi/12, 11\pi/12, 19\pi/12, 23\pi/12$

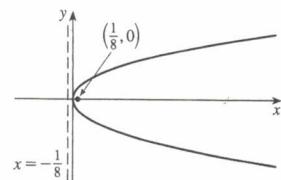
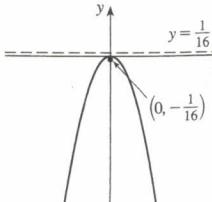
41. $(\frac{1}{2}\sqrt{3}, \pi/3), (\frac{1}{2}\sqrt{3}, 2\pi/3)$, and the pole43. Intersection at $\theta \approx 0.89, 2.25$; area ≈ 3.46 45. π 47. $\frac{8}{3}[(\pi^2 + 1)^{3/2} - 1]$

49. 29.0653

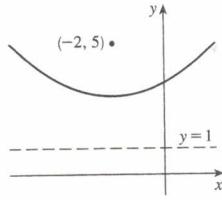
51. 9.6884

53. $\frac{16}{3}$ 

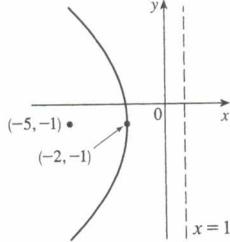
EXERCISES 11.5 ■ PAGE 696

1. $(0, 0), (\frac{1}{8}, 0), x = -\frac{1}{8}$ 3. $(0, 0), (0, -\frac{1}{16}), y = \frac{1}{16}$ 

5. $(-2, 3), (-2, 5)$, $y = 1$

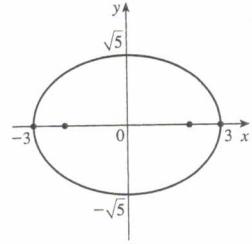


7. $(-2, -1), (-5, -1)$, $x = 1$

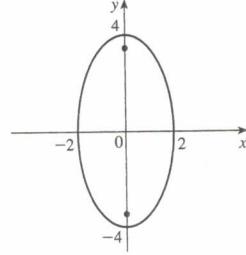


9. $x = -y^2$, focus $(-\frac{1}{4}, 0)$, directrix $x = \frac{1}{4}$

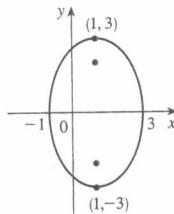
11. $(\pm 3, 0), (\pm 2, 0)$



13. $(0, \pm 4), (0, \pm 2\sqrt{3})$

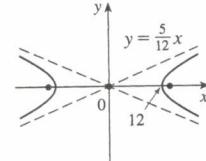


15. $(1, \pm 3), (1, \pm \sqrt{5})$



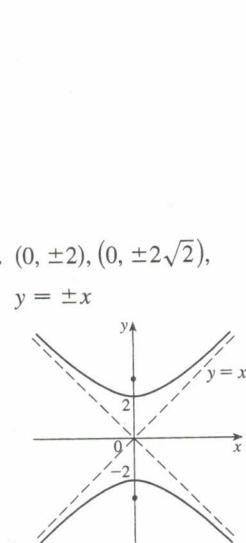
19. $(\pm 12, 0), (\pm 13, 0)$,

$y = \pm \frac{5}{12}x$

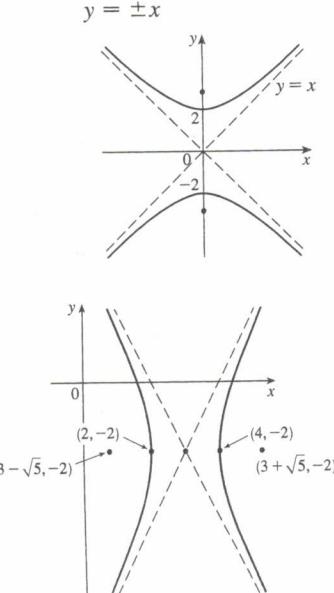


23. $(4, -2), (2, -2)$;
 $(3 \pm \sqrt{5}, -2)$;
 $y + 2 = \pm 2(x - 3)$

17. $\frac{x^2}{4} + \frac{y^2}{9} = 1$, foci $(0, \pm \sqrt{5})$



21. $(0, \pm 2), (0, \pm 2\sqrt{2})$,



25. Parabola, $(0, -1), (0, -\frac{3}{4})$

27. Ellipse, $(\pm\sqrt{2}, 1), (\pm 1, 1)$

29. Hyperbola, $(0, 1), (0, -3); (0, -1 \pm \sqrt{5})$

31. $x^2 = -8y$

33. $y^2 = -12(x + 1)$

35. $y - 3 = 2(x - 2)^2$

37. $\frac{x^2}{25} + \frac{y^2}{21} = 1$ 39. $\frac{x^2}{12} + \frac{(y - 4)^2}{16} = 1$

41. $\frac{(x + 1)^2}{12} + \frac{(y - 4)^2}{16} = 1$ 43. $\frac{x^2}{9} - \frac{y^2}{16} = 1$

45. $\frac{(y - 1)^2}{25} - \frac{(x + 3)^2}{39} = 1$ 47. $\frac{x^2}{9} - \frac{y^2}{36} = 1$

49. $\frac{x^2}{3,763,600} + \frac{y^2}{3,753,196} = 1$

51. (a) $\frac{121x^2}{1,500,625} - \frac{121y^2}{3,339,375} = 1$ (b) ≈ 248 mi

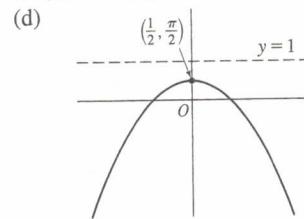
55. (a) Ellipse (b) Hyperbola (c) No curve

59. 9.69 61. $\frac{b^2c}{a} + ab \ln\left(\frac{a}{b+c}\right)$ where $c^2 = a^2 + b^2$

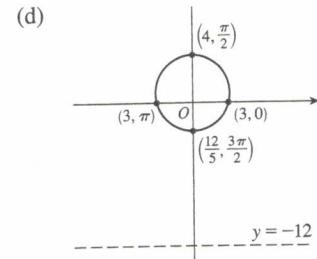
EXERCISES 11.6 ■ PAGE 704

1. $r = \frac{42}{4 + 7 \sin \theta}$ 3. $r = \frac{15}{4 - 3 \cos \theta}$

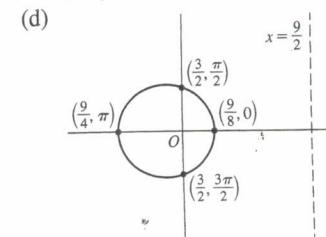
5. $r = \frac{8}{1 - \sin \theta}$ 7. $r = \frac{4}{2 + \cos \theta}$

9. (a) 1 (b) Parabola (c) $y = 1$ 

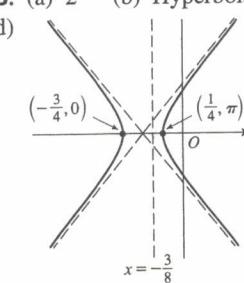
11. (a) $\frac{1}{4}$ (b) Ellipse (c) $y = -12$



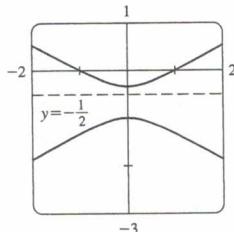
13. (a) $\frac{1}{3}$ (b) Ellipse (c) $x = \frac{9}{2}$



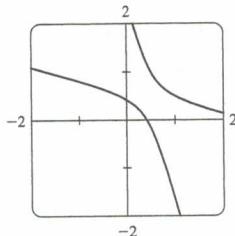
15. (a) 2 (b) Hyperbola (c) $x = -\frac{3}{8}$
 (d)



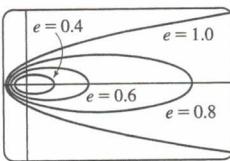
17. (a) $2, y = -\frac{1}{2}$



(b) $r = \frac{1}{1 - 2 \sin(\theta - 3\pi/4)}$



19. The ellipse is nearly circular when e is close to 0 and becomes more elongated as $e \rightarrow 1^-$. At $e = 1$, the curve becomes a parabola.



25. $r = \frac{2.26 \times 10^8}{1 + 0.093 \cos \theta}$

27. 35.64 AU 29. 7.0×10^7 km

31. 3.6×10^8 km

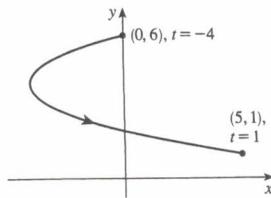
CHAPTER II REVIEW ■ PAGE 705

True-False Quiz

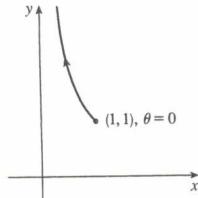
1. False 3. False 5. True 7. False 9. True

Exercises

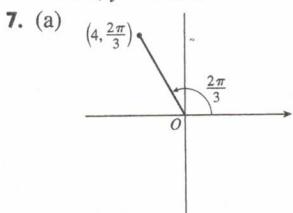
1. $x = y^2 - 8y + 12$



3. $y = 1/x$

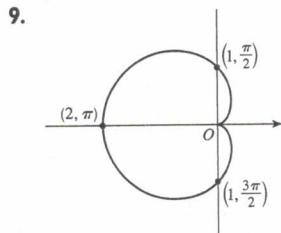


5. $x = t, y = \sqrt{t}; x = t^4, y = t^2;$
 $x = \tan^2 t, y = \tan t, 0 \leq t < \pi/2$

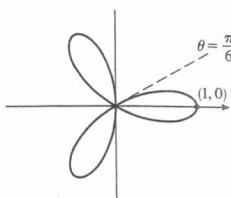


- (b) $(3\sqrt{2}, 3\pi/4),$
 $(-3\sqrt{2}, 7\pi/4)$

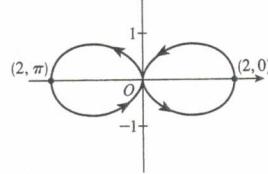
9. $(-2, 2\sqrt{3})$



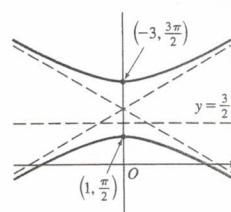
11.



13.

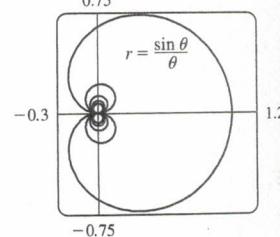


15.



17. $r = \frac{2}{\cos \theta + \sin \theta}$

19.



21. 2 23. -1

25. $\frac{1 + \sin t}{1 + \cos t}, \frac{1 + \cos t + \sin t}{(1 + \cos t)^3}$

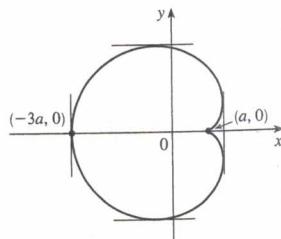
27. $(\frac{11}{8}, \frac{3}{4})$

29. Vertical tangent at

$(\frac{3}{2}a, \pm \frac{1}{2}\sqrt{3}a), (-3a, 0);$

horizontal tangent at

$(a, 0), (-\frac{1}{2}a, \pm \frac{3}{2}\sqrt{3}a)$



31. 18 33. $(2, \pm \pi/3)$ 35. $\frac{1}{2}(\pi - 1)$

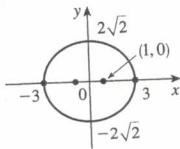
37. $2(5\sqrt{5} - 1)$

39. $\frac{2\sqrt{\pi^2 + 1} - \sqrt{4\pi^2 + 1}}{2\pi} + \ln\left(\frac{2\pi + \sqrt{4\pi^2 + 1}}{\pi + \sqrt{\pi^2 + 1}}\right)$

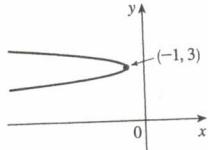
41. $471,295\pi/1024$

43. All curves have the vertical asymptote $x = 1$. For $c < -1$, the curve bulges to the right. At $c = -1$, the curve is the line $x = 1$. For $-1 < c < 0$, it bulges to the left. At $c = 0$ there is a cusp at $(0, 0)$. For $c > 0$, there is a loop.

45. $(\pm 1, 0), (\pm 3, 0)$



47. $(-\frac{25}{24}, 3), (-1, 3)$



49. $\frac{x^2}{25} + \frac{y^2}{9} = 1$

51. $\frac{y^2}{72/5} - \frac{x^2}{8/5} = 1$

53. $\frac{x^2}{25} + \frac{(8y - 399)^2}{160,801} = 1$

55. $r = \frac{4}{3 + \cos \theta}$

57. $x = a(\cot \theta + \sin \theta \cos \theta), y = a(1 + \sin^2 \theta)$

PROBLEMS PLUS ■ PAGE 708

1. $\ln(\pi/2)$

3. $[-\frac{3}{4}\sqrt{3}, \frac{3}{4}\sqrt{3}] \times [-1, 2]$

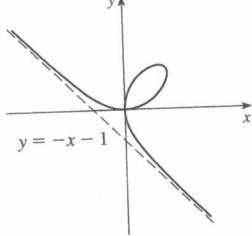
5. (a) At $(0, 0)$ and $(\frac{3}{2}, \frac{3}{2})$

(b) Horizontal tangents at $(0, 0)$ and $(\sqrt[3]{2}, \sqrt[3]{4})$

vertical tangents at $(0, 0)$ and $(\sqrt[3]{4}, \sqrt[3]{2})$

(g) $\frac{3}{2}$

(d)



57. $-1 < r < 1$

59. Convergent by the Monotonic Sequence Theorem; $5 \leq L < 8$

61. Decreasing; yes

63. Not monotonic; no

65. Decreasing; yes

67. 2 69. $\frac{1}{2}(3 + \sqrt{5})$

71. (b) $\frac{1}{2}(1 + \sqrt{5})$

73. (a) 0 (b) 9, 11

EXERCISES 12.2 ■ PAGE 730

1. (a) A sequence is an ordered list of numbers whereas a series is the sum of a list of numbers.

(b) A series is convergent if the sequence of partial sums is a convergent sequence. A series is divergent if it is not convergent.

3. $-2.40000, -1.92000,$

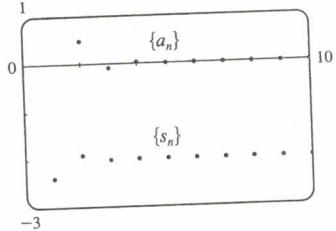
$-2.01600, -1.99680,$

$-2.00064, -1.99987,$

$-2.00003, -1.99999,$

$-2.00000, -2.00000;$

convergent, sum = -2



5. $1.55741, -0.62763,$

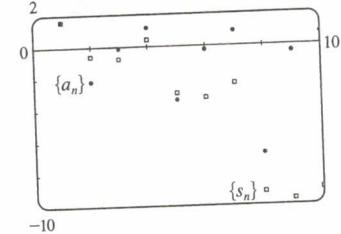
$-0.77018, 0.38764,$

$-2.99287, -3.28388,$

$-2.41243, -9.21214,$

$-9.66446, -9.01610;$

divergent



7. $0.29289, 0.42265,$

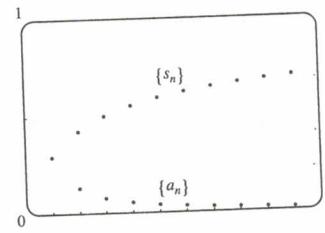
$0.50000, 0.55279,$

$0.59175, 0.62204,$

$0.64645, 0.66667,$

$0.68377, 0.69849;$

convergent, sum = 1



CHAPTER 12

EXERCISES 12.1 ■ PAGE 720

Abbreviations: C, convergent; D, divergent

1. (a) A sequence is an ordered list of numbers. It can also be defined as a function whose domain is the set of positive integers.
(b) The terms a_n approach 8 as n becomes large.
(c) The terms a_n become large as n becomes large.

3. 0.8, 0.96, 0.992, 0.9984, 0.99968 5. $-3, \frac{3}{2}, -\frac{1}{2}, \frac{1}{8}, -\frac{1}{40}$
7. 3, 5, 9, 17, 33 9. $a_n = 1/(2n - 1)$ 11. $a_n = 5n - 3$
13. $a_n = (-\frac{2}{3})^{n-1}$ 15. $\frac{1}{3}, \frac{2}{5}, \frac{3}{7}, \frac{4}{9}, \frac{5}{11}, \frac{6}{13}$; yes; $\frac{1}{2}$
17. 1 19. 5 21. 1 23. 1 25. 0 27. D
29. 0 31. 0 33. 0 35. 0 37. 1 39. e^2
41. $\ln 2$ 43. D 45. D 47. 1 49. $\frac{1}{2}$
51. D 53. 0
55. (a) 1060, 1123.60, 1191.02, 1262.48, 1338.23 (b) D

9. (a) C (b) D 11. 9 13. D 15. 60 17. $\frac{1}{7}$

19. D 21. D 23. D 25. $\frac{5}{2}$ 27. D 29. D

31. D 33. $e/(e - 1)$ 35. $\frac{3}{2}$ 37. $\frac{11}{6}$ 39. $e - 1$

41. $\frac{2}{9}$ 43. 1138/333 45. 5063/3300

47. $-3 < x < 3; \frac{x}{3-x}$ 49. $-\frac{1}{4} < x < \frac{1}{4}; \frac{1}{1-4x}$

51. All $x; \frac{2}{2 - \cos x}$ 53. 1

55. $a_1 = 0, a_n = \frac{2}{n(n+1)}$ for $n > 1$, sum = 1

57. (a) $S_n = \frac{D(1 - c^n)}{1 - c}$ (b) 5 59. $\frac{1}{2}(\sqrt{3} - 1)$

63. $\frac{1}{n(n+1)}$ 65. The series is divergent.

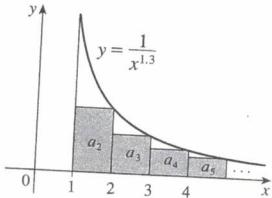
71. $\{s_n\}$ is bounded and increasing.

73. (a) $0, \frac{1}{9}, \frac{2}{9}, \frac{1}{3}, \frac{2}{3}, \frac{7}{9}, \frac{8}{9}, 1$

75. (a) $\frac{1}{2}, \frac{5}{6}, \frac{23}{24}, \frac{119}{120}; \frac{(n+1)! - 1}{(n+1)!}$ (c) 1

EXERCISES 12.3 ■ PAGE 739

I. C



3. D 5. C 7. C 9. D 11. C 13. D 15. C
 17. C 19. C 21. D 23. C 25. C 27. $p > 1$
 29. $p < -1$ 31. $(1, \infty)$
 33. (a) 1.54977, error ≤ 0.1 (b) 1.64522, error ≤ 0.005
 (c) $n > 1000$
 35. 0.00145 41. $b < 1/e$

EXERCISES 12.4 ■ PAGE 745

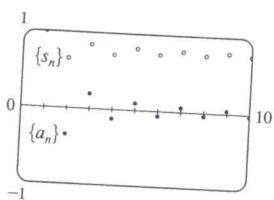
- I. (a) Nothing (b) C 3. C 5. D 7. C 9. C
 11. C 13. C 15. C 17. D 19. D 21. C
 23. C 25. D 27. C 29. C 31. D
 33. 1.249, error < 0.1 35. 0.76352, error < 0.001
 45. Yes

EXERCISES 12.5 ■ PAGE 749

I. (a) A series whose terms are alternately positive and negative (b) $0 < b_{n+1} \leq b_n$ and $\lim_{n \rightarrow \infty} b_n = 0$, where $b_n = |a_n|$ (c) $|R_n| \leq b_{n+1}$

3. C 5. C 7. D 9. C 11. C 13. D
 15. C 17. C 19. D

21. 1.0000, 0.6464,
 0.8389, 0.7139, 0.8033,
 0.7353, 0.7893, 0.7451, 0.7821,
 0.7505; error < 0.0275



23. 5 25. 4 27. 0.9721 29. 0.0676
 31. An underestimate 33. p is not a negative integer
 35. $\{b_n\}$ is not decreasing

EXERCISES 12.6 ■ PAGE 755

Abbreviations: AC, absolutely convergent;
 CC, conditionally convergent

- I. (a) D (b) C (c) May converge or diverge
 3. AC 5. CC 7. AC 9. D 11. AC 13. AC

15. AC 17. CC 19. AC 21. AC 23. D
 25. AC 27. D 29. D 31. (a) and (d)
 35. (a) $\frac{661}{960} \approx 0.68854$, error < 0.00521
 (b) $n \geq 11$, 0.693109

EXERCISES 12.7 ■ PAGE 758

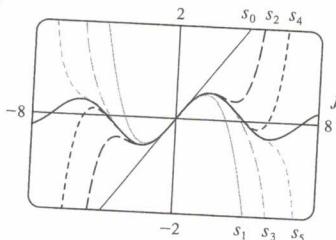
- I. C 3. D 5. C 7. D 9. C 11. C 13. C
 15. C 17. D 19. C 21. C 23. D 25. C
 27. C 29. C 31. D 33. C 35. C 37. C

EXERCISES 12.8 ■ PAGE 763

- I. A series of the form $\sum_{n=0}^{\infty} c_n(x-a)^n$, where x is a variable and a and the c_n 's are constants
 3. 1, $[-1, 1]$ 5. 1, $[-1, 1]$ 7. $\infty, (-\infty, \infty)$
 9. 2, $(-2, 2)$ 11. $\frac{1}{2}, (-\frac{1}{2}, \frac{1}{2})$ 13. 4, $(-4, 4)$
 15. 1, $[1, 3]$ 17. $\frac{1}{3}, [-\frac{13}{3}, -\frac{11}{3}]$ 19. $\infty, (-\infty, \infty)$
 21. $b, (a-b, a+b)$ 23. 0, $\{\frac{1}{2}\}$ 25. $\frac{1}{4}, [-\frac{1}{2}, 0]$
 27. $\infty, (-\infty, \infty)$ 29. (a) Yes (b) No 31. k^k 33. No

35. (a) $(-\infty, \infty)$

(b), (c)

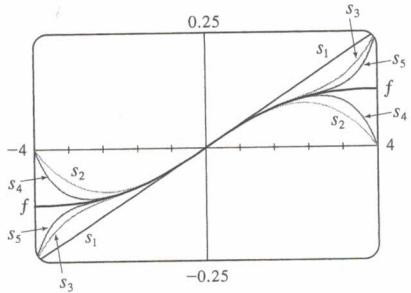


37. $(-1, 1), f(x) = (1 + 2x)/(1 - x^2)$ 41. 2

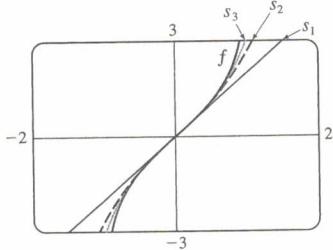
EXERCISES 12.9 ■ PAGE 769

- I. 10 3. $\sum_{n=0}^{\infty} (-1)^n x^n, (-1, 1)$ 5. $2 \sum_{n=0}^{\infty} \frac{1}{3^{n+1}} x^n, (-3, 3)$
 7. $\sum_{n=0}^{\infty} (-1)^n \frac{1}{9^{n+1}} x^{2n+1}, (-3, 3)$ 9. $1 + 2 \sum_{n=1}^{\infty} x^n, (-1, 1)$
 11. $\sum_{n=0}^{\infty} \left[(-1)^{n+1} - \frac{1}{2^{n+1}} \right] x^n, (-1, 1)$
 13. (a) $\sum_{n=0}^{\infty} (-1)^n (n+1)x^n, R = 1$
 (b) $\frac{1}{2} \sum_{n=0}^{\infty} (-1)^n (n+2)(n+1)x^n, R = 1$
 (c) $\frac{1}{2} \sum_{n=2}^{\infty} (-1)^n n(n-1)x^n, R = 1$
 15. $\ln 5 - \sum_{n=1}^{\infty} \frac{x^n}{n5^n}, R = 5$ 17. $\sum_{n=3}^{\infty} \frac{n-2}{2^{n-1}} x^n, R = 2$

19. $\sum_{n=0}^{\infty} (-1)^n \frac{1}{16^{n+1}} x^{2n+1}, R = 4$



21. $\sum_{n=0}^{\infty} \frac{2x^{2n+1}}{2n+1}, R = 1$



23. $C + \sum_{n=0}^{\infty} \frac{t^{8n+2}}{8n+2}, R = 1$

25. $C + \sum_{n=1}^{\infty} (-1)^{n+1} \frac{x^{2n-1}}{4n^2-1}, R = 1$

27. 0.199989 29. 0.000983 31. 0.09531
33. (b) 0.920 37. $[-1, 1], [-1, 1), (-1, 1)$

EXERCISES 12.10 ■ PAGE 782

1. $b_8 = f^{(8)}(5)/8!$ 3. $\sum_{n=0}^{\infty} (n+1)x^n, R = 1$

5. $\sum_{n=0}^{\infty} (n+1)x^n, R = 1$

7. $\sum_{n=0}^{\infty} (-1)^n \frac{\pi^{2n+1}}{(2n+1)!} x^{2n+1}, R = \infty$

9. $\sum_{n=0}^{\infty} \frac{5^n}{n!} x^n, R = \infty$ 11. $\sum_{n=0}^{\infty} \frac{x^{2n+1}}{(2n+1)!}, R = \infty$

13. $-1 - 2(x-1) + 3(x-1)^2 + 4(x-1)^3 + (x-1)^4, R = \infty$

15. $\sum_{n=0}^{\infty} \frac{e^3}{n!} (x-3)^n, R = \infty$

17. $\sum_{n=0}^{\infty} (-1)^{n+1} \frac{1}{(2n)!} (x-\pi)^{2n}, R = \infty$

19. $\frac{1}{3} + \sum_{n=1}^{\infty} (-1)^n \frac{1 \cdot 3 \cdot 5 \cdots (2n-1)}{2^n \cdot 3^{2n+1} \cdot n!} (x-9)^n, R = 9$

25. $1 + \frac{x}{2} + \sum_{n=2}^{\infty} (-1)^{n-1} \frac{1 \cdot 3 \cdot 5 \cdots (2n-3)}{2^n n!} x^n, R = 1$

27. $\sum_{n=0}^{\infty} (-1)^n \frac{(n+1)(n+2)}{2^{n+4}} x^n, R = 2$

29. $\sum_{n=0}^{\infty} (-1)^n \frac{\pi^{2n+1}}{(2n+1)!} x^{2n+1}, R = \infty$

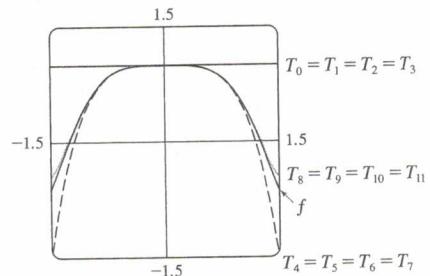
31. $\sum_{n=0}^{\infty} \frac{2^n+1}{n!} x^n, R = \infty$

33. $\sum_{n=0}^{\infty} (-1)^n \frac{1}{2^{2n}(2n)!} x^{4n+1}, R = \infty$

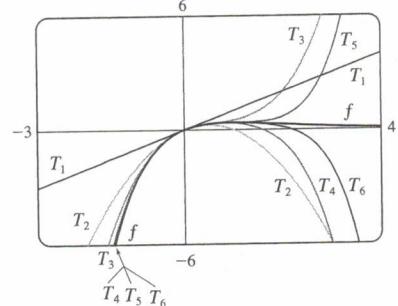
35. $\frac{1}{2}x + \sum_{n=1}^{\infty} (-1)^n \frac{1 \cdot 3 \cdot 5 \cdots (2n-1)}{n! 2^{3n+1}} x^{2n+1}, R = 2$

37. $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{2^{2n-1}}{(2n)!} x^{2n}, R = \infty$

39. $\sum_{n=0}^{\infty} (-1)^n \frac{1}{(2n)!} x^{4n}, R = \infty$



41. $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{(n-1)!} x^n, R = \infty$



43. 0.81873

45. (a) $1 + \sum_{n=1}^{\infty} \frac{1 \cdot 3 \cdot 5 \cdots (2n-1)}{2^n n!} x^{2n}$

(b) $x + \sum_{n=1}^{\infty} \frac{1 \cdot 3 \cdot 5 \cdots (2n-1)}{(2n+1)2^n n!} x^{2n+1}$

47. $C + \sum_{n=0}^{\infty} (-1)^n \frac{x^{6n+2}}{(6n+2)(2n)!}, R = \infty$

49. $C + \sum_{n=1}^{\infty} (-1)^n \frac{1}{2n(2n)!} x^{2n}, R = \infty$ 51. 0.440

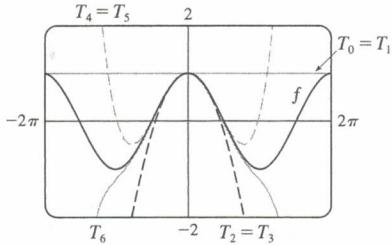
53. 0.40102 55. $\frac{1}{3}$ 57. $\frac{1}{120}$ 59. $1 - \frac{3}{2}x^2 + \frac{25}{24}x^4$

61. $1 + \frac{1}{6}x^2 + \frac{7}{360}x^4$ 63. e^{-x^4}

65. $1/\sqrt{2^x}$ 67. $e^3 - 1$

EXERCISES 12.11 ■ PAGE 791

1. (a) $T_0(x) = 1 = T_1(x)$, $T_2(x) = 1 - \frac{1}{2}x^2 = T_3(x)$,
 $T_4(x) = 1 - \frac{1}{2}x^2 + \frac{1}{24}x^4 = T_5(x)$,
 $T_6(x) = 1 - \frac{1}{2}x^2 + \frac{1}{24}x^4 - \frac{1}{720}x^6$

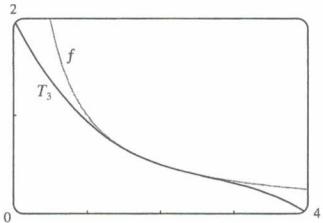


(b)

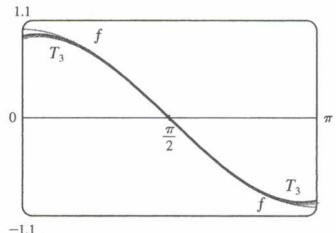
x	f	$T_0 = T_1$	$T_2 = T_3$	$T_4 = T_5$	T_6
$\frac{\pi}{4}$	0.7071	1	0.6916	0.7074	0.7071
$\frac{\pi}{2}$	0	1	-0.2337	0.0200	-0.0009
π	-1	1	-3.9348	0.1239	-1.2114

(c) As n increases, $T_n(x)$ is a good approximation to $f(x)$ on a larger and larger interval.

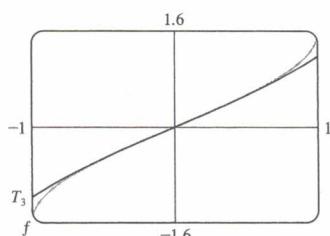
3. $\frac{1}{2} - \frac{1}{4}(x-2) + \frac{1}{8}(x-2)^2 - \frac{1}{16}(x-2)^3$



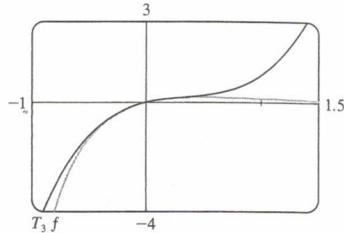
5. $-\left(x - \frac{\pi}{2}\right) + \frac{1}{6}\left(x - \frac{\pi}{2}\right)^3$



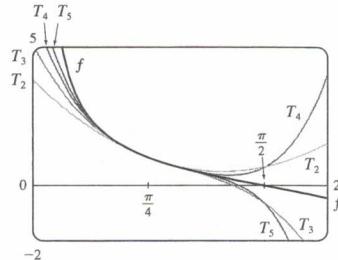
7. $x + \frac{1}{6}x^3$



9. $x - 2x^2 + 2x^3$



11. $T_5(x) = 1 - 2\left(x - \frac{\pi}{4}\right) + 2\left(x - \frac{\pi}{4}\right)^2 - \frac{8}{3}\left(x - \frac{\pi}{4}\right)^3 + \frac{10}{3}\left(x - \frac{\pi}{4}\right)^4 - \frac{64}{15}\left(x - \frac{\pi}{4}\right)^5$



13. (a) $2 + \frac{1}{4}(x-4) - \frac{1}{64}(x-4)^2$ (b) 1.5625×10^{-5}
 15. (a) $1 + \frac{2}{3}(x-1) - \frac{1}{9}(x-1)^2 + \frac{4}{81}(x-1)^3$ (b) 0.000097
 17. (a) $1 + \frac{1}{2}x^2$ (b) 0.0015 19. (a) $1 + x^2$ (b) 0.00006
 21. (a) $x^2 - \frac{1}{6}x^4$ (b) 0.042 23. 0.17365 25. Four
 27. $-1.037 < x < 1.037$ 29. $-0.86 < x < 0.86$
 31. 21 m, no 37. (c) They differ by about 8×10^{-9} km.

CHAPTER 12 REVIEW ■ PAGE 795

True-False Quiz

1. False 3. True 5. False 7. False
 9. False 11. True 13. True 15. False
 17. True 19. True

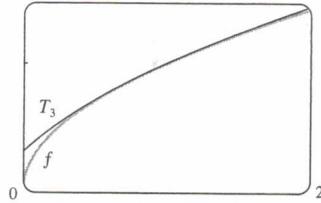
Exercises

1. $\frac{1}{2}$ 3. D 5. 0 7. e^{12} 9. 2 11. C
 13. C 15. D 17. C 19. C 21. C 23. CC
 25. AC 27. $\frac{1}{11}$ 29. $\pi/4$ 31. e^{-e} 35. 0.9721
 37. 0.18976224, error $< 6.4 \times 10^{-7}$
 41. 4, $[-6, 2)$ 43. 0.5, $[2.5, 3.5)$
 45. $\frac{1}{2} \sum_{n=0}^{\infty} (-1)^n \left[\frac{1}{(2n)!} \left(x - \frac{\pi}{6}\right)^{2n} + \frac{\sqrt{3}}{(2n+1)!} \left(x - \frac{\pi}{6}\right)^{2n+1} \right]$
 47. $\sum_{n=0}^{\infty} (-1)^n x^{n+2}$, $R = 1$ 49. $-\sum_{n=1}^{\infty} \frac{x^n}{n}$, $R = 1$
 51. $\sum_{n=0}^{\infty} (-1)^n \frac{x^{8n+4}}{(2n+1)!}$, $R = \infty$
 53. $\frac{1}{2} + \sum_{n=1}^{\infty} \frac{1 \cdot 5 \cdot 9 \cdots (4n-3)}{n! 2^{6n+1}} x^n$, $R = 16$

55. $C + \ln|x| + \sum_{n=1}^{\infty} \frac{x^n}{n \cdot n!}$

57. (a) $1 + \frac{1}{2}(x-1) - \frac{1}{8}(x-1)^2 + \frac{1}{16}(x-1)^3$

(b) 1.5 (c) 0.000006



59. $-\frac{1}{6}$

PROBLEMS PLUS ■ PAGE 797

1. $15!/5! = 10,897,286,400$

3. (b) 0 if $x = 0$, $(1/x) - \cot x$ if $x \neq k\pi$, k an integer
5. (a) $s_n = 3 \cdot 4^n$, $l_n = 1/3^n$, $p_n = 4^n/3^{n-1}$ (c) $\frac{2}{5}\sqrt{3}$

9. $(-1, 1)$, $\frac{x^3 + 4x^2 + x}{(1-x)^4}$ II. $\ln \frac{1}{2}$

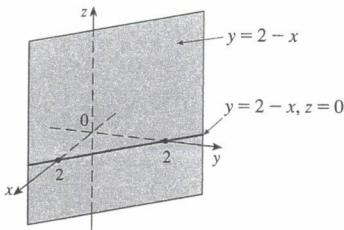
13. (a) $\frac{250}{101}\pi(e^{-(n-1)\pi/5} - e^{-n\pi/5})$ (b) $\frac{250}{101}\pi$

CHAPTER 13

EXERCISES 13.1 ■ PAGE 805

1. $(4, 0, -3)$ 3. $Q; R$

5. A vertical plane that intersects the xy -plane in the line $y = 2 - x$, $z = 0$ (see graph at right)



7. $|PQ| = 6$, $|QR| = 2\sqrt{10}$, $|RP| = 6$; isosceles triangle

9. (a) No (b) Yes

11. $(x-1)^2 + (y+4)^4 + (z-3)^2 = 25$;
 $(x-1)^2 + (z-3)^2 = 9$, $y = 0$ (a circle)

13. $(x-3)^2 + (y-8)^2 + (z-1)^2 = 30$

15. $(3, -2, 1)$, 5

17. $(2, 0, -6)$, $9/\sqrt{2}$ 19. (b) $\frac{5}{2}, \frac{1}{2}\sqrt{94}, \frac{1}{2}\sqrt{85}$

21. (a) $(x-2)^2 + (y+3)^2 + (z-6)^2 = 36$

(b) $(x-2)^2 + (y+3)^2 + (z-6)^2 = 4$

(c) $(x-2)^2 + (y+3)^2 + (z-6)^2 = 9$

23. A plane parallel to the xz -plane and 4 units to the left of it

25. A half-space consisting of all points in front of the plane $x = 3$

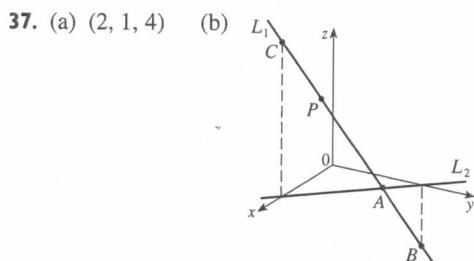
27. All points on or between the horizontal planes $z = 0$ and $z = 6$

29. All points on or inside a sphere with radius $\sqrt{3}$ and center O

31. All points on or inside a circular cylinder of radius 3 with axis the y -axis

33. $0 < x < 5$ 35. $r^2 < x^2 + y^2 + z^2 < R^2$

37. (a) $(2, 1, 4)$



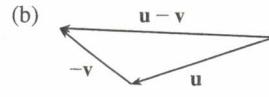
39. $14x - 6y - 10z = 9$, a plane perpendicular to AB

EXERCISES 13.2 ■ PAGE 813

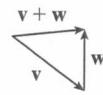
1. (a) Scalar (b) Vector (c) Vector (d) Scalar

3. $\vec{AB} = \vec{DC}$, $\vec{DA} = \vec{CB}$, $\vec{DE} = \vec{EB}$, $\vec{EA} = \vec{CE}$

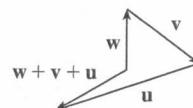
5. (a)



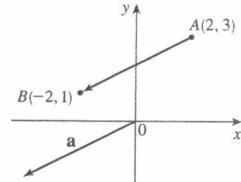
(c)



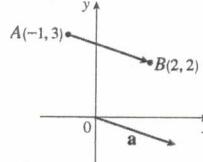
(d)



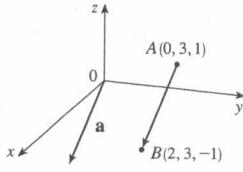
7. $\mathbf{a} = \langle -4, -2 \rangle$



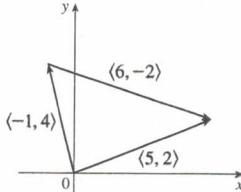
9. $\mathbf{a} = \langle 3, -1 \rangle$



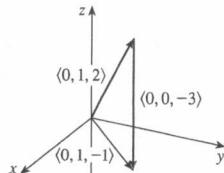
11. $\mathbf{a} = \langle 2, 0, -2 \rangle$



13. $\langle 5, 2 \rangle$



15. $\langle 0, 1, -1 \rangle$



17. $\langle 2, -18 \rangle$, $\langle 1, -42 \rangle$, 13, 10

19. $-\mathbf{i} + \mathbf{j} + 2\mathbf{k}$, $-4\mathbf{i} + \mathbf{j} + 9\mathbf{k}$, $\sqrt{14}$, $\sqrt{82}$

21. $-\frac{3}{\sqrt{58}}\mathbf{i} + \frac{7}{\sqrt{58}}\mathbf{j}$ 23. $\frac{8}{9}\mathbf{i} - \frac{1}{9}\mathbf{j} + \frac{4}{9}\mathbf{k}$

25. $\langle 2, 2\sqrt{3} \rangle$ 27. $\approx 45.96 \text{ ft/s}$, $\approx 38.57 \text{ ft/s}$

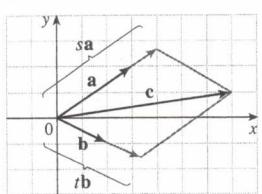
29. $100\sqrt{7} \approx 264.6 \text{ N}$, $\approx 139.1^\circ$

31. $\sqrt{493} \approx 22.2 \text{ mi/h}$, N 8° W

33. $\mathbf{T}_1 \approx -196\mathbf{i} + 3.92\mathbf{j}$, $\mathbf{T}_2 \approx 196\mathbf{i} + 3.92\mathbf{j}$

35. $\pm(\mathbf{i} + 4\mathbf{j})/\sqrt{17}$ 37. 0

39. (a), (b)



(d) $s = \frac{9}{7}$, $t = \frac{11}{7}$

41. A sphere with radius 1, centered at (x_0, y_0, z_0)

EXERCISES 13.3 ■ PAGE 820

1. (b), (c), (d) are meaningful

3. 14 5. 19 7. 32 9. -15

11. $\mathbf{u} \cdot \mathbf{v} = \frac{1}{2}$, $\mathbf{u} \cdot \mathbf{w} = -\frac{1}{2}$

15. $\cos^{-1}\left(\frac{9 - 4\sqrt{7}}{20}\right) \approx 95^\circ$ 17. $\cos^{-1}\left(\frac{5}{\sqrt{1015}}\right) \approx 81^\circ$

19. $\cos^{-1}\left(\frac{-1}{2\sqrt{7}}\right) \approx 101^\circ$ 21. $45^\circ, 45^\circ, 90^\circ$

23. (a) Neither (b) Orthogonal
(c) Orthogonal (d) Parallel25. Yes 27. $\langle \mathbf{i} - \mathbf{j} - \mathbf{k} \rangle/\sqrt{3}$ [or $\langle -\mathbf{i} + \mathbf{j} + \mathbf{k} \rangle/\sqrt{3}$]

29. $\frac{3}{5\sqrt{2}}, \frac{4}{5\sqrt{2}}, \frac{1}{\sqrt{2}}$; $65^\circ, 56^\circ, 45^\circ$

31. $\frac{2}{7}, \frac{3}{7}, -\frac{6}{7}$; $73^\circ, 65^\circ, 149^\circ$

33. $1/\sqrt{3}, 1/\sqrt{3}, 1/\sqrt{3}$; $55^\circ, 55^\circ, 55^\circ$

35. 3, $\langle \frac{9}{5}, -\frac{12}{5} \rangle$ 37. $\frac{9}{7}, \langle \frac{27}{49}, \frac{54}{49}, -\frac{18}{49} \rangle$

39. $1/\sqrt{21}, \frac{2}{21}\mathbf{i} - \frac{1}{21}\mathbf{j} + \frac{4}{21}\mathbf{k}$

43. $\langle 0, 0, -2\sqrt{10} \rangle$ or any vector of the form $\langle s, t, 3s - 2\sqrt{10} \rangle$, $s, t \in \mathbb{R}$

45. 144 J 47. $2400 \cos(40^\circ) \approx 1839 \text{ ft-lb}$ 49. $\frac{13}{5}$

51. $\cos^{-1}(1/\sqrt{3}) \approx 55^\circ$

EXERCISES 13.4 ■ PAGE 828

1. $16\mathbf{i} + 48\mathbf{k}$ 3. $15\mathbf{i} - 3\mathbf{j} + 3\mathbf{k}$ 5. $\frac{1}{2}\mathbf{i} - \mathbf{j} + \frac{3}{2}\mathbf{k}$

7. $t^4\mathbf{i} - 2t^3\mathbf{j} + t^2\mathbf{k}$ 9. 0 11. $\mathbf{i} + \mathbf{j} + \mathbf{k}$

13. (a) Scalar (b) Meaningless (c) Vector
(d) Meaningless (e) Meaningless (f) Scalar15. 24; into the page 17. $\langle 5, -3, 1 \rangle, \langle -5, 3, -1 \rangle$

19. $\langle -2/\sqrt{6}, -1/\sqrt{6}, 1/\sqrt{6} \rangle, \langle 2/\sqrt{6}, 1/\sqrt{6}, -1/\sqrt{6} \rangle$

27. 16 29. (a) $\langle 6, 3, 2 \rangle$ (b) $\frac{7}{2}$

31. (a) $\langle 13, -14, 5 \rangle$ (b) $\frac{1}{2}\sqrt{390}$

33. 82 35. 3 39. $10.8 \sin 80^\circ \approx 10.6 \text{ N} \cdot \text{m}$

41. $\approx 417 \text{ N}$ 43. (b) $\sqrt{97/3}$

49. (a) No (b) No (c) Yes

EXERCISES 13.5 ■ PAGE 838

1. (a) True (b) False (c) True (d) False (e) False
(f) True (g) False (h) True (i) True (j) False
(k) True

3. $\mathbf{r} = (2\mathbf{i} + 2.4\mathbf{j} + 3.5\mathbf{k}) + t(3\mathbf{i} + 2\mathbf{j} - \mathbf{k})$;

$x = 2 + 3t, y = 2.4 + 2t, z = 3.5 - t$

5. $\mathbf{r} = (\mathbf{i} + 6\mathbf{k}) + t(\mathbf{i} + 3\mathbf{j} + \mathbf{k})$;

$x = 1 + t, y = 3t, z = 6 + t$

7. $x = 1 - 5t, y = 3, z = 2 - 2t; \frac{x-1}{-5} = \frac{z-2}{-2}, y = 3$

9. $x = 2 + 2t, y = 1 + \frac{1}{2}t, z = -3 - 4t$;

$(x-2)/2 = 2y-2 = (z+3)/(-4)$

11. $x = 1 + t, y = -1 + 2t, z = 1 + t$;
 $x-1 = (y+1)/2 = z-1$

13. Yes

15. (a) $(x-1)/(-1) = (y+5)/2 = (z-6)/(-3)$

(b) $(-1, -1, 0), (-\frac{3}{2}, 0, -\frac{3}{2}), (0, -3, 3)$

17. $\mathbf{r}(t) = (2\mathbf{i} - \mathbf{j} + 4\mathbf{k}) + t(2\mathbf{i} + 7\mathbf{j} - 3\mathbf{k}), 0 \leq t \leq 1$

19. Parallel 21. Skew

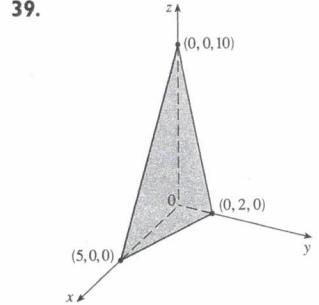
23. $-2x + y + 5z = 1$ 25. $x + y - z = -1$

27. $2x - y + 3z = 0$ 29. $3x - 7z = -9$

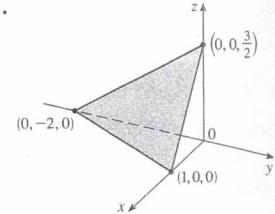
31. $x + y + z = 2$ 33. $-13x + 17y + 7z = -42$

35. $33x + 10y + 4z = 190$ 37. $x - 2y + 4z = -1$

39.



41.



43. (2, 3, 5) 45. (2, 3, 1) 47. 1, 0, -1

49. Perpendicular 51. Neither, $\approx 70.5^\circ$ 53. Parallel

55. (a) $x = 1, y = -t, z = t$ (b) $\cos^{-1}\left(\frac{5}{3\sqrt{3}}\right) \approx 15.8^\circ$

57. $x = 1, y - 2 = -z$

59. $x + 2y + z = 5$ 61. $(x/a) + (y/b) + (z/c) = 1$

63. $x = 3t, y = 1 - t, z = 2 - 2t$

65. P_1 and P_3 are parallel, P_2 and P_4 are identical

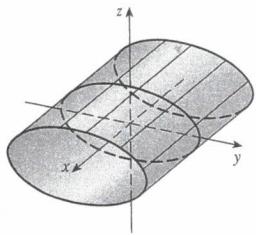
67. $\sqrt{61/14}$ 69. $\frac{18}{7}$ 71. $5/(2\sqrt{14})$ 75. $1/\sqrt{6}$

EXERCISES 13.6 ■ PAGE 846

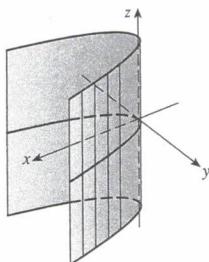
1. (a) Parabola

(b) Parabolic cylinder with rulings parallel to the z -axis(c) Parabolic cylinder with rulings parallel to the x -axis

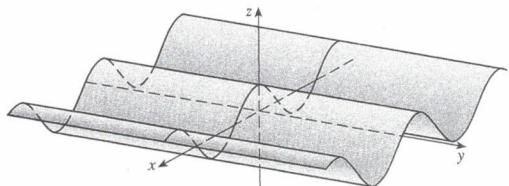
3. Elliptic cylinder



5. Parabolic cylinder

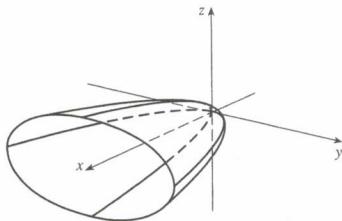


7. Cylindrical surface

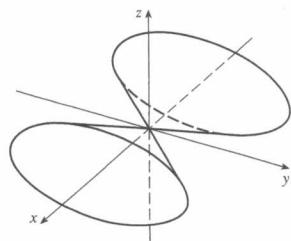


9. (a) $x = k$, $y^2 - z^2 = 1 - k^2$, hyperbola ($k \neq \pm 1$);
 $y = k$, $x^2 - z^2 = 1 - k^2$, hyperbola ($k \neq \pm 1$);
 $z = k$, $x^2 + y^2 = 1 + k^2$, circle
(b) The hyperboloid is rotated so that it has axis the y-axis
(c) The hyperboloid is shifted one unit in the negative y-direction

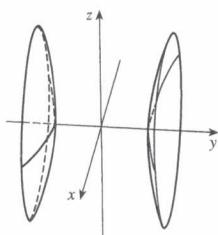
11. Elliptic paraboloid with axis the x-axis



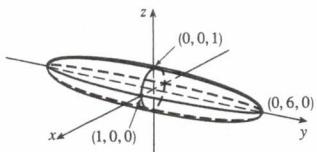
13. Elliptic cone with axis the x-axis



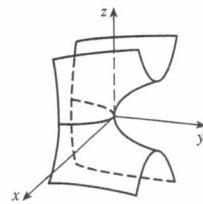
15. Hyperboloid of two sheets



17. Ellipsoid



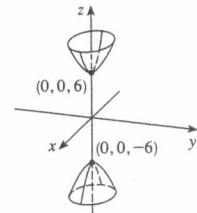
19. Hyperbolic paraboloid



21. VII 23. II 25. VI 27. VIII

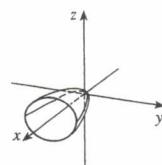
29. $-\frac{x^2}{9} - \frac{y^2}{4} + \frac{z^2}{36} = 1$

Hyperboloid of two sheets
with axis the z-axis



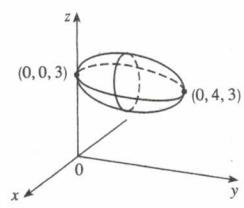
31. $\frac{x^2}{6} = \frac{y^2}{3} + \frac{z^2}{2}$

Elliptic paraboloid with vertex
(0, 0, 0) and axis the x-axis

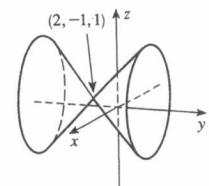


33. $x^2 + \frac{(y-2)^2}{4} + (z-3)^2 = 1$

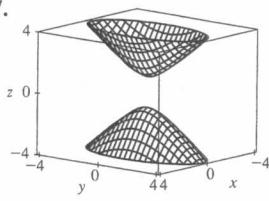
Ellipsoid with center (0, 2, 3)



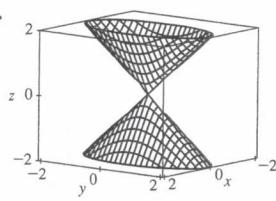
35. $(y+1)^2 = (x-2)^2 + (z-1)^2$
Circular cone with vertex (2, -1, 1)
and axis parallel to the y-axis



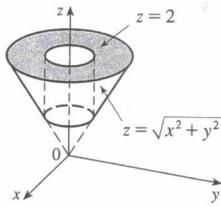
37.



39.



41.



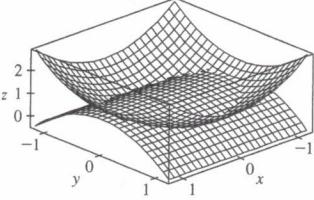
43. $y = x^2 + z^2$

45. $-4x = y^2 + z^2$, paraboloid

47. (a) $\frac{x^2}{(6378.137)^2} + \frac{y^2}{(6378.137)^2} + \frac{z^2}{(6356.523)^2} = 1$

(b) Circle (c) Ellipse

51.



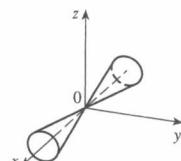
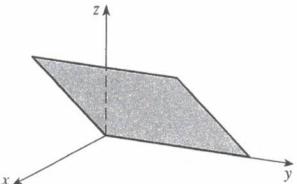
CHAPTER 13 REVIEW ■ PAGE 848

True-False Quiz

- I. True 3. True 5. True 7. True 9. True
II. False 13. False 15. False 17. True

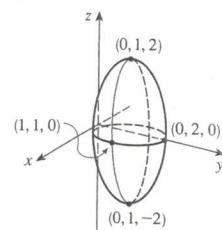
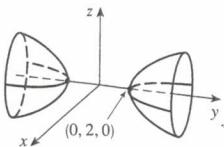
Exercises

1. (a) $(x + 1)^2 + (y - 2)^2 + (z - 1)^2 = 69$
(b) $(y - 2)^2 + (z - 1)^2 = 68$, $x = 0$
(c) Center $(4, -1, -3)$, radius 5
3. $\mathbf{u} \cdot \mathbf{v} = 3\sqrt{2}$; $|\mathbf{u} \times \mathbf{v}| = 3\sqrt{2}$; out of the page
5. $-2, -4$ 7. (a) 2 (b) -2 (c) -2 (d) 0
9. $\cos^{-1}\left(\frac{1}{3}\right) \approx 71^\circ$ 11. (a) $\langle 4, -3, 4 \rangle$ (b) $\sqrt{41}/2$
13. 166 N, 114 N
15. $x = 4 - 3t$, $y = -1 + 2t$, $z = 2 + 3t$
17. $x = -2 + 2t$, $y = 2 - t$, $z = 4 + 5t$
19. $-4x + 3y + z = -14$ 21. $(1, 4, 4)$
23. Skew 25. $x + y + z = 4$
27. $22/\sqrt{26}$
29. Plane 31. Cone



33. Hyperboloid of two sheets

35. Ellipsoid



37. $4x^2 + y^2 + z^2 = 16$

PROBLEMS PLUS ■ PAGE 851

1. $(\sqrt{3} - 1.5) \text{ m}$

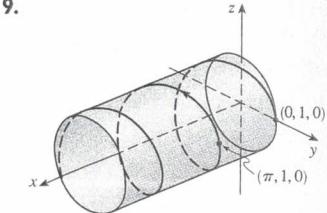
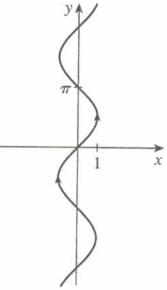
3. (a) $(x + 1)/(-2c) = (y - c)/(c^2 - 1) = (z - c)/(c^2 + 1)$
(b) $x^2 + y^2 = t^2 + 1$, $z = t$ (c) $4\pi/3$

CHAPTER 14

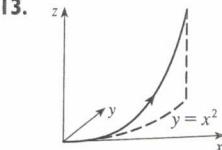
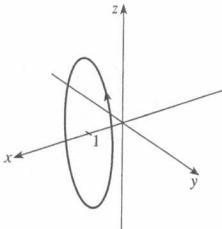
EXERCISES 14.1 ■ PAGE 858

1. $(-1, 2]$ 3. $\langle 1, 0, 0 \rangle$ 5. $\mathbf{i} + \mathbf{j} + \mathbf{k}$

7.



11.

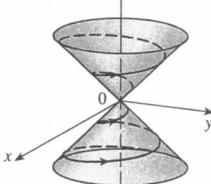


15. $\mathbf{r}(t) = \langle t, 2t, 3t \rangle$, $0 \leq t \leq 1$;
 $x = t$, $y = 2t$, $z = 3t$, $0 \leq t \leq 1$

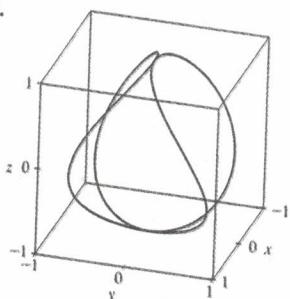
17. $\mathbf{r}(t) = \langle 3t + 1, 2t - 1, 5t + 2 \rangle$, $0 \leq t \leq 1$;
 $x = 3t + 1$, $y = 2t - 1$, $z = 5t + 2$, $0 \leq t \leq 1$

19. VI 21. IV 23. V

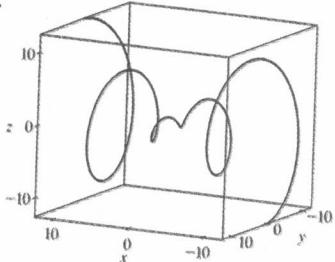
25. 27. $(0, 0, 0), (1, 0, 1)$



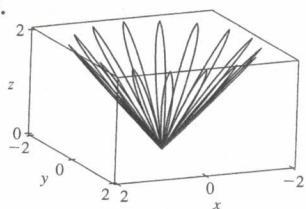
29.



31.



33.

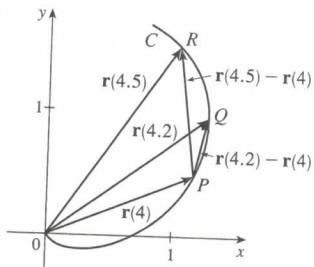


37. $\mathbf{r}(t) = t\mathbf{i} + \frac{1}{2}(t^2 - 1)\mathbf{j} + \frac{1}{2}(t^2 + 1)\mathbf{k}$

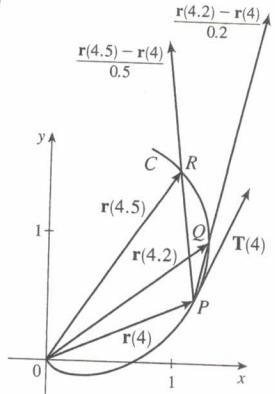
39. $x = 2 \cos t, y = 2 \sin t, z = 4 \cos^2 t$ 41. Yes

EXERCISES 14.2 ■ PAGE 864

1. (a)

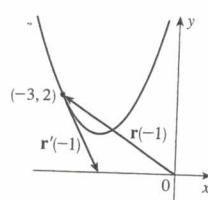


(b), (d)



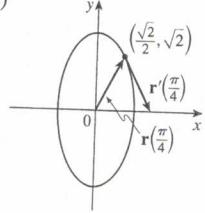
(c) $\mathbf{r}'(4) = \lim_{h \rightarrow 0} \frac{\mathbf{r}(4+h) - \mathbf{r}(4)}{h}; \mathbf{T}(4) = \frac{\mathbf{r}'(4)}{|\mathbf{r}'(4)|}$

3. (a), (c)

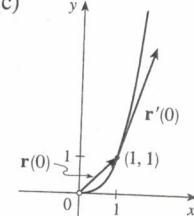


(b) $\mathbf{r}'(t) = \langle 1, 2t \rangle$

5. (a), (c)



7. (a), (c)



(b) $\mathbf{r}'(t) = \cos t \mathbf{i} - 2 \sin t \mathbf{j}$ (b) $\mathbf{r}'(t) = e^t \mathbf{i} + 3e^{3t} \mathbf{j}$

9. $\mathbf{r}'(t) = \langle t \cos t + \sin t, 2t, \cos 2t - 2t \sin 2t \rangle$

11. $\mathbf{r}'(t) = 4e^{4t} \mathbf{k}$ 13. $\mathbf{r}'(t) = 2te^{t^2} \mathbf{i} + [3/(1+3t)] \mathbf{k}$

15. $\mathbf{r}'(t) = \mathbf{b} + 2t\mathbf{c}$ 17. $\langle \frac{1}{3}, \frac{2}{3}, \frac{2}{3} \rangle$ 19. $\frac{3}{5}\mathbf{j} + \frac{4}{5}\mathbf{k}$

21. $\langle 1, 2t, 3t^2 \rangle, \langle 1/\sqrt{14}, 2/\sqrt{14}, 3/\sqrt{14} \rangle, \langle 0, 2, 6t \rangle, \langle 6t^2, -6t, 2 \rangle$

23. $x = 3 + t, y = 2t, z = 2 + 4t$

25. $x = 1 - t, y = t, z = 1 - t$

27. $x = t, y = 1 - t, z = 2t$

29. $x = -\pi - t, y = \pi + t, z = -\pi t$

31. 66° 33. $4\mathbf{i} - 3\mathbf{j} + 5\mathbf{k}$ 35. $\mathbf{i} + \mathbf{j} + \mathbf{k}$

37. $e^t \mathbf{i} + t^2 \mathbf{j} + (t \ln t - t) \mathbf{k} + \mathbf{C}$

39. $t^2 \mathbf{i} + t^3 \mathbf{j} + (\frac{2}{3}t^{3/2} - \frac{2}{3}) \mathbf{k}$

45. $2t \cos t + 2 \sin t - 2 \cos t \sin t$

EXERCISES 14.3 ■ PAGE 872

1. $20\sqrt{29}$

3. $e - e^{-1}$

9. 1.2780

5. $\frac{1}{27}(13^{3/2} - 8)$

7. 15.3841

13. $\mathbf{r}(t(s)) = \frac{2}{\sqrt{29}}s \mathbf{i} + \left(1 - \frac{3}{\sqrt{29}}s\right) \mathbf{j} + \left(5 + \frac{4}{\sqrt{29}}s\right) \mathbf{k}$

15. $(3 \sin 1, 4, 3 \cos 1)$

17. (a) $\langle (2/\sqrt{29}) \cos t, 5/\sqrt{29}, (-2/\sqrt{29}) \sin t \rangle, \langle -\sin t, 0, -\cos t \rangle$ (b) $\frac{2}{29}$

19. (a) $\frac{1}{e^{2t} + 1} \langle \sqrt{2}e^t, e^{2t}, -1 \rangle, \frac{1}{e^{2t} + 1} \langle 1 - e^{2t}, \sqrt{2}e^t, \sqrt{2}e^t \rangle$

(b) $\sqrt{2}e^{2t}/(e^{2t} + 1)^2$

21. $2/(4t^2 + 1)^{3/2}$

23. $\frac{4}{25}$

25. $\frac{1}{7}\sqrt{\frac{19}{14}}$

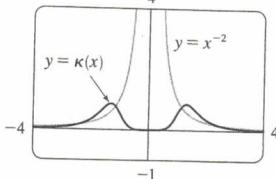
27. $2/(4x^2 - 8x + 5)^{3/2}$

29. $15\sqrt{x}/(1 + 100x^3)^{3/2}$

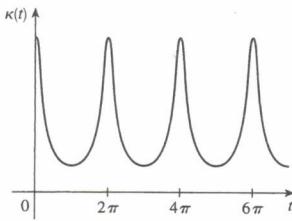
31. $(-\frac{1}{2} \ln 2, 1/\sqrt{2})$; approaches 0

33. (a) P (b) $1.3, 0.7$

35.


 37. a is $y = f(x)$, b is $y = \kappa(x)$

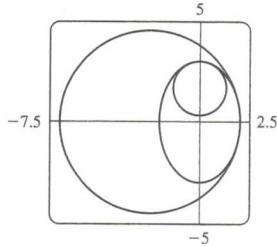
39. $\kappa(t) = \frac{6\sqrt{4\cos^2 t - 12\cos t + 13}}{(17 - 12\cos t)^{3/2}}$

integer multiples of 2π

41. $1/(\sqrt{2}e^t)$ 43. $\left\langle \frac{2}{3}, \frac{2}{3}, \frac{1}{3} \right\rangle, \left\langle -\frac{1}{3}, \frac{2}{3}, -\frac{2}{3} \right\rangle, \left\langle -\frac{2}{3}, \frac{1}{3}, \frac{2}{3} \right\rangle$

45. $y = 6x + \pi, x + 6y = 6\pi$

47. $(x + \frac{5}{2})^2 + y^2 = \frac{81}{4}, x^2 + (y - \frac{5}{3})^2 = \frac{16}{9}$



49. $(-1, -3, 1)$ 57. $2/(t^4 + 4t^2 + 1)$

59. $2.07 \times 10^{10} \text{ \AA} \approx 2 \text{ m}$

EXERCISES 14.4 ■ PAGE 882

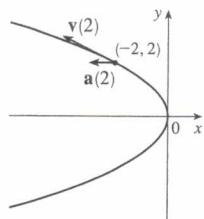
1. (a) $1.8\mathbf{i} - 3.8\mathbf{j} - 0.7\mathbf{k}, 2.0\mathbf{i} - 2.4\mathbf{j} - 0.6\mathbf{k}, 2.8\mathbf{i} + 1.8\mathbf{j} - 0.3\mathbf{k}, 2.8\mathbf{i} + 0.8\mathbf{j} - 0.4\mathbf{k}$

(b) $2.4\mathbf{i} - 0.8\mathbf{j} - 0.5\mathbf{k}, 2.58$

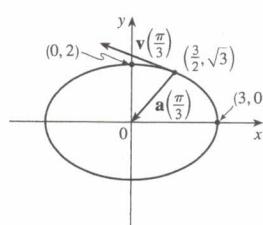
3. $\mathbf{v}(t) = \langle -t, 1 \rangle$

$\mathbf{a}(t) = \langle -1, 0 \rangle$

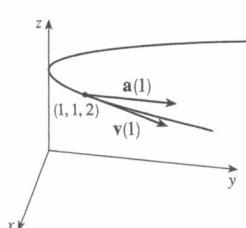
$|\mathbf{v}(t)| = \sqrt{t^2 + 1}$



5. $\mathbf{v}(t) = -3 \sin t \mathbf{i} + 2 \cos t \mathbf{j}$
 $\mathbf{a}(t) = -3 \cos t \mathbf{i} - 2 \sin t \mathbf{j}$
 $|\mathbf{v}(t)| = \sqrt{5 \sin^2 t + 4}$



7. $\mathbf{v}(t) = \mathbf{i} + 2t\mathbf{j}$
 $\mathbf{a}(t) = 2\mathbf{j}$
 $|\mathbf{v}(t)| = \sqrt{1 + 4t^2}$



9. $\langle 2t, 3t^2, 2t \rangle, \langle 2, 6t, 2 \rangle, |t| \sqrt{9t^2 + 8}$

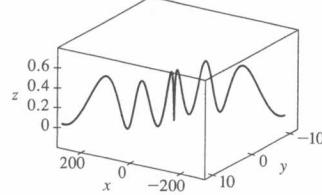
11. $\sqrt{2}\mathbf{i} + e^t\mathbf{j} - e^{-t}\mathbf{k}, e^t\mathbf{j} + e^{-t}\mathbf{k}, e^t + e^{-t}$

13. $e^t[(\cos t - \sin t)\mathbf{i} + (\sin t + \cos t)\mathbf{j} + (t + 1)\mathbf{k}], e^t\sqrt{t^2 + 2t + 3}$

15. $\mathbf{v}(t) = t\mathbf{i} + 2t\mathbf{j} + \mathbf{k}, \mathbf{r}(t) = (\frac{1}{2}t^2 + 1)\mathbf{i} + t^2\mathbf{j} + t\mathbf{k}$

17. (a) $\mathbf{r}(t) = (\frac{1}{3}t^3 + t)\mathbf{i} + (t - \sin t + 1)\mathbf{j} + (\frac{1}{4} - \frac{1}{4}\cos 2t)\mathbf{k}$

(b)



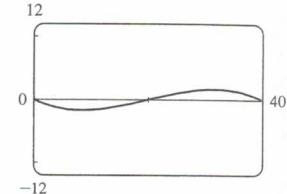
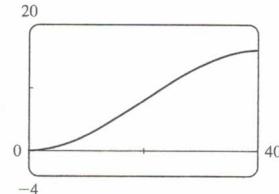
19. $t = 4$ 21. $\mathbf{r}(t) = t\mathbf{i} - t\mathbf{j} + \frac{5}{2}t^2\mathbf{k}, |\mathbf{v}(t)| = \sqrt{25t^2 + 2}$

23. (a) $\approx 22 \text{ km}$ (b) $\approx 3.2 \text{ km}$ (c) 500 m/s

25. 30 m/s 27. $\approx 10.2^\circ, \approx 79.8^\circ$

29. $13.0^\circ < \theta < 36.0^\circ, 55.4^\circ < \theta < 85.5^\circ$

31. (a) 16 m (b) $\approx 23.6^\circ$ upstream



33. $6t, 6$ 35. $0, 1$ 37. $e^t - e^{-t}, \sqrt{2}$

39. $4.5 \text{ cm/s}^2, 9.0 \text{ cm/s}^2$ 41. $t = 1$

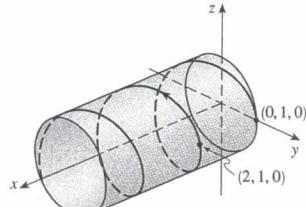
CHAPTER 14 REVIEW ■ PAGE 886

True-False Quiz

1. True 3. False 5. False 7. True
 9. False 11. True

Exercises

1. (a)



(b) $\mathbf{r}'(t) = \mathbf{i} - \pi \sin \pi t \mathbf{j} + \pi \cos \pi t \mathbf{k}, \mathbf{r}''(t) = -\pi^2 \cos \pi t \mathbf{j} - \pi^2 \sin \pi t \mathbf{k}$

3. $\mathbf{r}(t) = 4 \cos t \mathbf{i} + 4 \sin t \mathbf{j} + (5 - 4 \cos t) \mathbf{k}, 0 \leq t \leq 2\pi$

5. $\frac{1}{3}\mathbf{i} - (2/\pi^2)\mathbf{j} + (2/\pi)\mathbf{k}$ 7. 86.631 9. $\pi/2$

11. (a) $\langle t^2, t, 1 \rangle / \sqrt{t^4 + t^2 + 1}$

(b) $\langle 2t, 1 - t^4, -2t^3 - t \rangle / \sqrt{t^8 + 4t^6 + 2t^4 + 5t^2}$

(c) $\sqrt{t^8 + 4t^6 + 2t^4 + 5t^2} / (t^4 + t^2 + 1)^2$

13. $12/17^{3/2}$ 15. $\dot{x} - 2y + 2\pi = 0$

17. $\mathbf{v}(t) = (1 + \ln t)\mathbf{i} + \mathbf{j} - e^{-t}\mathbf{k},$

$|\mathbf{v}(t)| = \sqrt{2 + 2 \ln t + (\ln t)^2 + e^{-2t}}, \mathbf{a}(t) = (1/t)\mathbf{i} + e^{-t}\mathbf{k}$

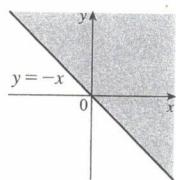
- 19.** (a) About 3.8 ft above the ground, 60.8 ft from the athlete
 (b) ≈ 21.4 ft (c) ≈ 64.2 ft from the athlete
21. (c) $-2e^{-t} \mathbf{v}_d + e^{-t} \mathbf{R}$

PROBLEMS PLUS ■ PAGE 888

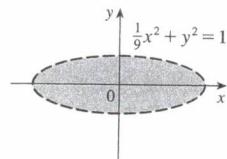
- 1.** (a) $\mathbf{v} = \omega R(-\sin \omega t \mathbf{i} + \cos \omega t \mathbf{j})$ (c) $\mathbf{a} = -\omega^2 \mathbf{r}$
3. (a) $90^\circ, v_0^2/(2g)$
5. (a) ≈ 0.94 ft to the right of the table's edge, ≈ 15 ft/s
 (b) $\approx 7.6^\circ$ (c) ≈ 2.13 ft to the right of the table's edge
7. 56°

CHAPTER 15**EXERCISES 15.1 ■ PAGE 901**

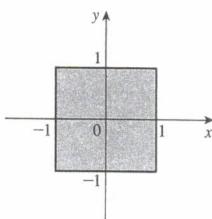
- 1.** (a) -27 ; a temperature of -15°C with wind blowing at 40 km/h feels equivalent to about -27°C without wind.
 (b) When the temperature is -20°C , what wind speed gives a wind chill of -30°C ? 20 km/h
 (c) With a wind speed of 20 km/h , what temperature gives a wind chill of -49°C ? -35°C
 (d) A function of wind speed that gives wind-chill values when the temperature is -5°C
 (e) A function of temperature that gives wind-chill values when the wind speed is 50 km/h
3. Yes
5. (a) 25 ; a 40-knot wind blowing in the open sea for 15 h will create waves about 25 ft high.
 (b) $f(30, t)$ is a function of t giving the wave heights produced by 30-knot winds blowing for t hours.
 (c) $f(v, 30)$ is a function of v giving the wave heights produced by winds of speed v blowing for 30 hours.
7. (a) 4 (b) \mathbb{R}^2 (c) $[0, \infty)$
9. (a) e (b) $\{(x, y, z) | z \geq x^2 + y^2\}$ (c) $[1, \infty)$
11. $\{(x, y) | y \geq -x\}$



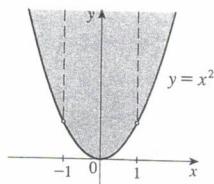
13. $\{(x, y) | \frac{1}{9}x^2 + y^2 < 1\}$



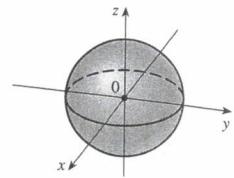
15. $\{(x, y) | -1 \leq x \leq 1, -1 \leq y \leq 1\}$



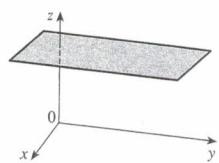
17. $\{(x, y) | y \geq x^2, x \neq \pm 1\}$



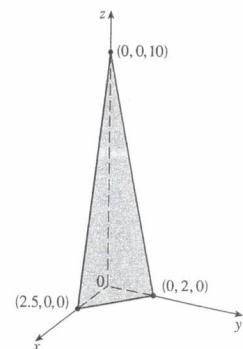
19. $\{(x, y, z) | x^2 + y^2 + z^2 \leq 1\}$



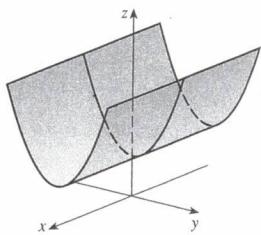
21. $z = 3$, horizontal plane



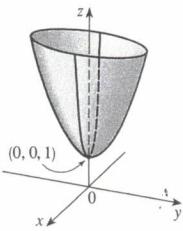
23. $4x + 5y + z = 10$, plane



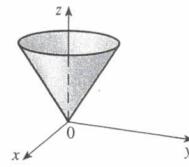
25. $z = y^2 + 1$, parabolic cylinder



27. $z = 4x^2 + y^2 + 1$
 elliptic paraboloid

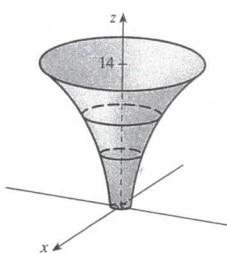


29. $z = \sqrt{x^2 + y^2}$,
 top half of cone

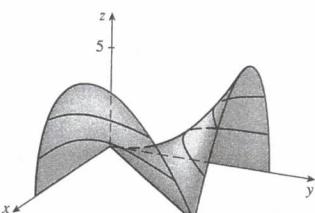


31. $\approx 56, \approx 35$ **33.** Steep; nearly flat

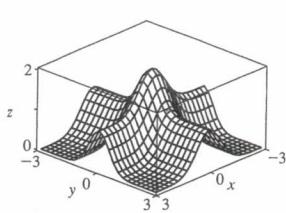
35.



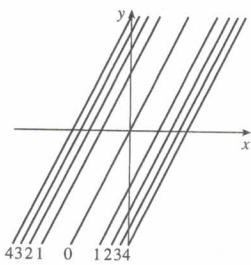
37.



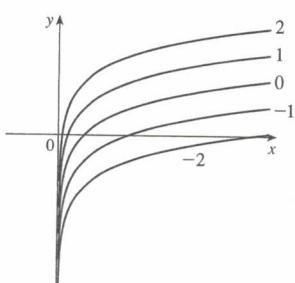
51.



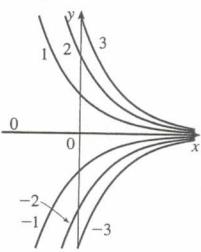
39. $(y - 2x)^2 = k$



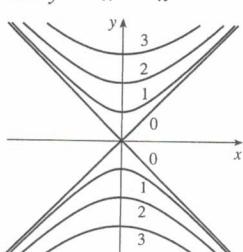
41. $y = \ln x + k$



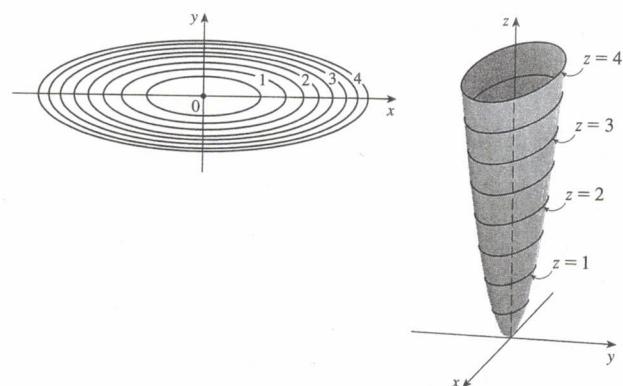
43. $y = ke^{-x}$



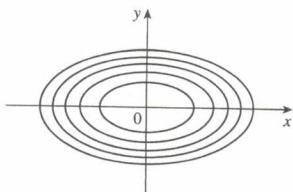
45. $y^2 - x^2 = k^2$



47. $x^2 + 9y^2 = k$



49.



55. (a) C (b) II 57. (a) F (b) I

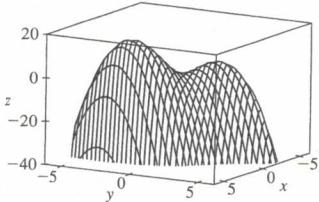
59. (a) B (b) VI

61. Family of parallel planes

63. Family of hyperboloids of one or two sheets with axis the y-axis

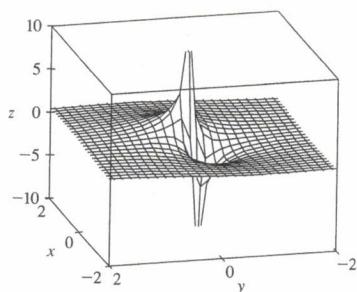
65. (a) Shift the graph of f upward 2 units
 (b) Stretch the graph of f vertically by a factor of 2
 (c) Reflect the graph of f about the xy -plane
 (d) Reflect the graph of f about the xy -plane and then shift it upward 2 units

67.



f appears to have a maximum value of about 15. There are two local maximum points but no local minimum point.

69.



The function values approach 0 as x, y become large; as (x, y) approaches the origin, f approaches $\pm\infty$ or 0, depending on the direction of approach.

71. If $c = 0$, the graph is a cylindrical surface. For $c > 0$, the level curves are ellipses. The graph curves upward as we leave the origin, and the steepness increases as c increases. For $c < 0$, the level curves are hyperbolas. The graph curves upward in the y -direction and downward in the x -direction, approaching the xy -plane, in the x -direction giving a saddle-shaped appearance near $(0, 0, 1)$.

- 73.
- $c = -2, 0, 2$
75. (b)
- $y = 0.75x + 0.01$

EXERCISES 15.2 ■ PAGE 913

1. Nothing; if f is continuous, $f(3, 1) = 6$ 3. $-\frac{5}{2}$
 5. 1 7. $\frac{2}{7}$ 9. Does not exist 11. Does not exist
 13. 0 15. Does not exist 17. 2 19. 1
 21. Does not exist

23. The graph shows that the function approaches different numbers along different lines.

$$25. h(x, y) = (2x + 3y - 6)^2 + \sqrt{2x + 3y - 6}; \\ \{(x, y) \mid 2x + 3y \geq 6\}$$

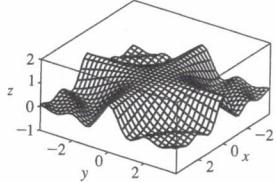
$$27. \text{Along the line } y = x \quad 29. \{(x, y) \mid y \neq \pm e^{x/2}\}$$

$$31. \{(x, y) \mid y \geq 0\} \quad 33. \{(x, y) \mid x^2 + y^2 > 4\}$$

$$35. \{(x, y, z) \mid y \geq 0, y \neq \sqrt{x^2 + z^2}\}$$

$$37. \{(x, y) \mid (x, y) \neq (0, 0)\} \quad 39. 0 \quad 41. -1$$

43.



f is continuous on \mathbb{R}^2

EXERCISES 15.3 ■ PAGE 924

1. (a) The rate of change of temperature as longitude varies, with latitude and time fixed; the rate of change as only latitude varies; the rate of change as only time varies.
 (b) Positive, negative, positive

3. (a) $f_T(-15, 30) \approx 1.3$; for a temperature of -15°C and wind speed of 30 km/h, the wind-chill index rises by 1.3°C for each degree the temperature increases. $f_v(-15, 30) \approx -0.15$; for a temperature of -15°C and wind speed of 30 km/h, the wind-chill index decreases by 0.15°C for each km/h the wind speed increases.

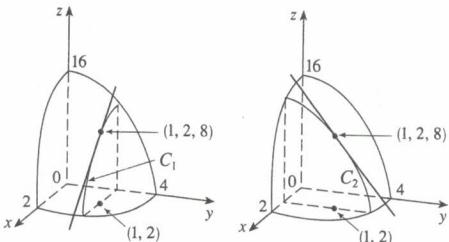
- (b) Positive, negative (c) 0

5. (a) Positive (b) Negative

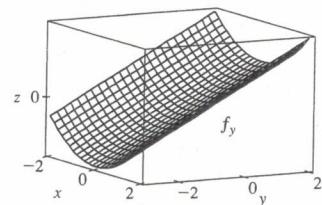
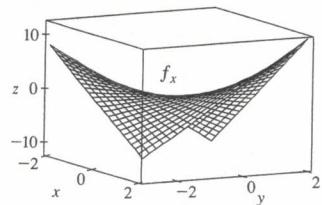
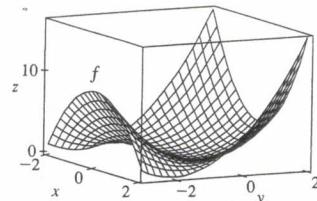
7. (a) Positive (b) Negative

9. $c = f$, $b = f_s$, $a = f_y$

11. $f_x(1, 2) = -8 = \text{slope of } C_1$, $f_y(1, 2) = -4 = \text{slope of } C_2$



$$13. f_x = 2x + 2xy, f_y = 2y + x^2$$



$$15. f_x(x, y) = -3y, f_y(x, y) = 5y^4 - 3x$$

$$17. f_x(x, t) = -\pi e^{-t} \sin \pi x, f_t(x, t) = -e^{-t} \cos \pi x$$

$$19. \frac{\partial z}{\partial x} = 20(2x + 3y)^9, \frac{\partial z}{\partial y} = 30(2x + 3y)^9$$

$$21. f_x(x, y) = 2y/(x + y)^2, f_y(x, y) = -2x/(x + y)^2$$

$$23. \frac{\partial w}{\partial \alpha} = \cos \alpha \cos \beta, \frac{\partial w}{\partial \beta} = -\sin \alpha \sin \beta$$

$$25. f_r(r, s) = \frac{2r^2}{r^2 + s^2} + \ln(r^2 + s^2), f_s(r, s) = \frac{2rs}{r^2 + s^2}$$

$$27. \frac{\partial u}{\partial t} = e^{w/t}(1 - u/t), \frac{\partial u}{\partial w} = e^{w/t}$$

$$29. f_x = z - 10xy^3z^4, f_y = -15x^2y^2z^4, f_z = x - 20x^2y^3z^3$$

$$31. \frac{\partial w}{\partial x} = 1/(x + 2y + 3z), \frac{\partial w}{\partial y} = 2/(x + 2y + 3z), \frac{\partial w}{\partial z} = 3/(x + 2y + 3z)$$

$$33. \frac{\partial u}{\partial x} = y \sin^{-1}(yz), \frac{\partial u}{\partial y} = x \sin^{-1}(yz) + xyz/\sqrt{1 - y^2z^2}, \frac{\partial u}{\partial z} = xy^2/\sqrt{1 - y^2z^2}$$

$$35. f_x = yz^2 \tan(yt), f_y = xyz^2 t \sec^2(yt) + xz^2 \tan(yt), f_z = 2xyz \tan(yt), f_t = xy^2 z^2 \sec^2(yt)$$

$$37. \frac{\partial u}{\partial x_i} = x_i/\sqrt{x_1^2 + x_2^2 + \dots + x_n^2}$$

$$39. \frac{1}{5} \quad 41. \frac{1}{4}$$

$$43. f_x(x, y) = y^2 - 3x^2y, f_y(x, y) = 2xy - x^3$$

$$45. \frac{\partial z}{\partial x} = \frac{3yz - 2x}{2z - 3xy}, \frac{\partial z}{\partial y} = \frac{3xz - 2y}{2z - 3xy}$$

$$47. \frac{\partial z}{\partial x} = \frac{1 + y^2z^2}{1 + y + y^2z^2}, \frac{\partial z}{\partial y} = \frac{-z}{1 + y + y^2z^2}$$

$$49. (a) f'(x), g'(y) \quad (b) f'(x + y), f'(x + y)$$

$$51. f_{xx} = 6xy^5 + 24x^2y, f_{xy} = 15x^2y^4 + 8x^3 = f_{yx}, f_{yy} = 20x^3y^3$$

$$53. w_{uu} = v^2/(u^2 + v^2)^{3/2}, w_{uv} = -uv/(u^2 + v^2)^{3/2} = w_{vu}, w_{vv} = u^2/(u^2 + v^2)^{3/2}$$

$$55. z_{xx} = -2x/(1 + x^2)^2, z_{xy} = 0 = z_{yx}, z_{yy} = -2y/(1 + y^2)^2$$

61. $12xy, 72xy$

63. $24 \sin(4x + 3y + 2z), 12 \sin(4x + 3y + 2z)$

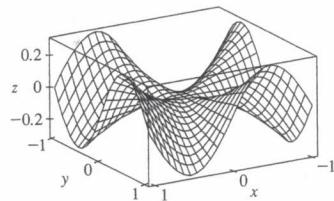
65. $\theta e^{r\theta}(2 \sin \theta + \theta \cos \theta + r\theta \sin \theta)$ 67. $4/(y + 2z)^3, 0$

69. $\approx 12.2, \approx 16.8, \approx 23.25$ 81. R^2/R_1^2

87. No 89. $x = 1 + t, y = 2, z = 2 - 2t$

93. -2

95. (a)



(b) $f_x(x, y) = \frac{x^4y + 4x^2y^3 - y^5}{(x^2 + y^2)^2}, f_y(x, y) = \frac{x^5 - 4x^3y^2 - xy^4}{(x^2 + y^2)^2}$

(c) 0, 0 (e) No, since f_{xy} and f_{yx} are not continuous.

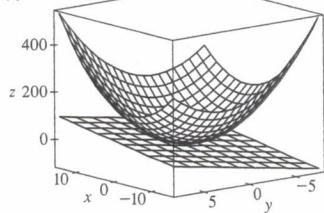
EXERCISES 15.4 ■ PAGE 935

1. $z = -8x - 2y$

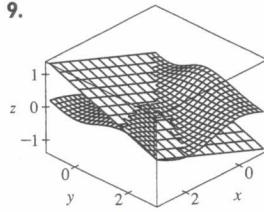
3. $x + y - 2z = 0$

5. $z = y$

7.



9.



11. $2x + \frac{1}{4}y - 1$ 13. $\frac{1}{9}x - \frac{2}{9}y + \frac{2}{3}$ 15. $1 - \pi y$

19. $-\frac{2}{3}x - \frac{7}{3}y + \frac{20}{3}; 2.846$ 21. $\frac{3}{7}x + \frac{2}{7}y + \frac{6}{7}z; 6.9914$

23. $4T + H - 329; 129^\circ F$

25. $dz = 3x^2 \ln(y^2) dx + (2x^3/y) dy$

27. $dm = 5p^4q^3 dp + 3p^5q^2 dq$

29. $dR = \beta^2 \cos \gamma d\alpha + 2\alpha\beta \cos \gamma d\beta - \alpha\beta^2 \sin \gamma d\gamma$

31. $\Delta z = 0.9225, dz = 0.9$ 33. 5.4 cm^2 35. 16 cm^3

37. 150 39. $\frac{1}{17} \approx 0.059 \Omega$ 41. 2.3%

43. $\varepsilon_1 = \Delta x, \varepsilon_2 = \Delta y$

EXERCISES 15.5 ■ PAGE 943

1. $(2x + y) \cos t + (2y + x)e^t$

3. $[(x/t) - y \sin t]/\sqrt{1 + x^2 + y^2}$

5. $e^{yz}[2t - (x/z) - (2xy/z^2)]$

7. $\frac{\partial z}{\partial s} = 2xy^3 \cos t + 3x^2y^2 \sin t,$

$\frac{\partial z}{\partial t} = -2sxy^3 \sin t + 3sx^2y^2 \cos t$

9. $\frac{\partial z}{\partial s} = t^2 \cos \theta \cos \phi - 2st \sin \theta \sin \phi,$

$\frac{\partial z}{\partial t} = 2st \cos \theta \cos \phi - s^2 \sin \theta \sin \phi$

11. $\frac{\partial z}{\partial s} = e^r \left(t \cos \theta - \frac{s}{\sqrt{s^2 + t^2}} \sin \theta \right),$

$\frac{\partial z}{\partial t} = e^r \left(s \cos \theta - \frac{t}{\sqrt{s^2 + t^2}} \sin \theta \right)$

13. 62 15. 7, 2

17. $\frac{\partial u}{\partial r} = \frac{\partial u}{\partial x} \frac{\partial x}{\partial r} + \frac{\partial u}{\partial y} \frac{\partial y}{\partial r}, \frac{\partial u}{\partial s} = \frac{\partial u}{\partial x} \frac{\partial x}{\partial s} + \frac{\partial u}{\partial y} \frac{\partial y}{\partial s},$
 $\frac{\partial u}{\partial t} = \frac{\partial u}{\partial x} \frac{\partial x}{\partial t} + \frac{\partial u}{\partial y} \frac{\partial y}{\partial t}$

19. $\frac{\partial w}{\partial x} = \frac{\partial w}{\partial r} \frac{\partial r}{\partial x} + \frac{\partial w}{\partial s} \frac{\partial s}{\partial x} + \frac{\partial w}{\partial t} \frac{\partial t}{\partial x},$
 $\frac{\partial w}{\partial y} = \frac{\partial w}{\partial r} \frac{\partial r}{\partial y} + \frac{\partial w}{\partial s} \frac{\partial s}{\partial y} + \frac{\partial w}{\partial t} \frac{\partial t}{\partial y}$

21. 85, 178, 54 23. $\frac{9}{7}, \frac{9}{7}$ 25. 36, 24, 30

27. $\frac{4(xy)^{3/2} - y}{x - 2x^2\sqrt{xy}}$ 29. $\frac{\sin(x - y) + e^y}{\sin(x - y) - xe^y}$

31. $\frac{3yz - 2x}{2z - 3xy}, \frac{3xz - 2y}{2z - 3xy}$

33. $\frac{1 + y^2z^2}{1 + y + y^2z^2}, -\frac{z}{1 + y + y^2z^2}$

35. $2^\circ C/s$ 37. $\approx -0.33 \text{ m/s per minute}$

39. (a) $6 \text{ m}^3/\text{s}$ (b) $10 \text{ m}^2/\text{s}$ (c) 0 m/s

41. $\approx -0.27 \text{ L/s}$ 43. $-1/(12\sqrt{3}) \text{ rad/s}$

45. (a) $\frac{\partial z}{\partial r} = (\frac{\partial z}{\partial x}) \cos \theta + (\frac{\partial z}{\partial y}) \sin \theta,$
 $\frac{\partial z}{\partial \theta} = -(\frac{\partial z}{\partial x})r \sin \theta + (\frac{\partial z}{\partial y})r \cos \theta$

51. $4rs \frac{\partial^2 z}{\partial x^2} + (4r^2 + 4s^2) \frac{\partial^2 z}{\partial x \partial y} + 4rs \frac{\partial^2 z}{\partial y^2} + 2 \frac{\partial z}{\partial y}$

EXERCISES 15.6 ■ PAGE 956

1. $\approx -0.08 \text{ mb/km}$ 3. ≈ 0.778 5. $2 + \sqrt{3}/2$

7. (a) $\nabla f(x, y) = \langle 2 \cos(2x + 3y), 3 \cos(2x + 3y) \rangle$

(b) $\langle 2, 3 \rangle$ (c) $\sqrt{3} - \frac{3}{2}$

9. (a) $\langle e^{2yz}, 2xze^{2yz}, 2xye^{2yz} \rangle$ (b) $\langle 1, 12, 0 \rangle$ (c) $-\frac{22}{3}$

11. $23/10$ 13. $-8/\sqrt{10}$ 15. $4/\sqrt{30}$ 17. $9/(2\sqrt{5})$

19. $2/5$ 21. $4\sqrt{2}, \langle -1, 1 \rangle$ 23. $1, \langle 0, 1 \rangle$

25. $1, \langle 3, 6, -2 \rangle$ 27. (b) $\langle -12, 92 \rangle$

29. All points on the line $y = x + 1$

31. (a) $-40/(3\sqrt{3})$

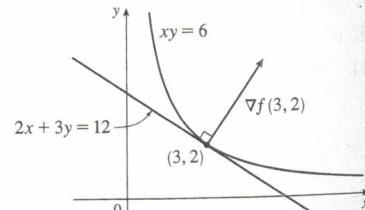
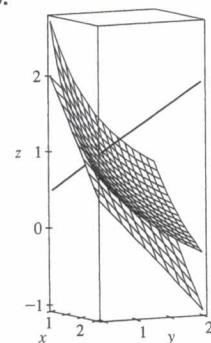
33. (a) $32/\sqrt{3}$ (b) $\langle 38, 6, 12 \rangle$ (c) $2\sqrt{406}$ 35. $\frac{327}{13}$

39. (a) $x + y + z = 11$ (b) $x - 3 = y - 3 = z - 5$

41. (a) $4x - 5y - z = 4$ (b) $\frac{x-2}{4} = \frac{y-1}{-5} = \frac{z+1}{-1}$

43. (a) $x + y - z = 1$ (b) $x - 1 = y = -z$

45. 47. $\langle 2, 3 \rangle, 2x + 3y = 12$

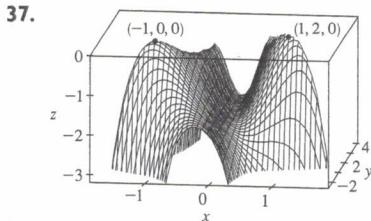


53. No 59. $x = -1 - 10t, y = 1 - 16t, z = 2 - 12t$

63. If $\mathbf{u} = \langle a, b \rangle$ and $\mathbf{v} = \langle c, d \rangle$, then $af_x + bf_y$ and $cf_x + df_y$ are known, so we solve linear equations for f_x and f_y .

EXERCISES 15.7 ■ PAGE 966

1. (a) f has a local minimum at $(1, 1)$.
 (b) f has a saddle point at $(1, 1)$.
 3. Local minimum at $(1, 1)$, saddle point at $(0, 0)$
 5. Maximum $f(-1, \frac{1}{2}) = 11$
 7. Minima $f(1, 1) = 0$, $f(-1, -1) = 0$, saddle point at $(0, 0)$
 9. Saddle points at $(1, -1)$, $(-1, 1)$
 11. Minimum $f(2, 1) = -8$, saddle point at $(0, 0)$
 13. None 15. Minimum $f(0, 0) = 0$, saddle points at $(\pm 1, 0)$
 17. Minima $f(0, 1) = f(\pi, -1) = f(2\pi, 1) = -1$, saddle points at $(\pi/2, 0)$, $(3\pi/2, 0)$
 21. Minima $f(1, \pm 1) = 3$, $f(-1, \pm 1) = 3$
 23. Maximum $f(\pi/3, \pi/3) = 3\sqrt{3}/2$, minimum $f(5\pi/3, 5\pi/3) = -3\sqrt{3}/2$, saddle point at (π, π)
 25. Minima $f(-1.714, 0) \approx -9.200$, $f(1.402, 0) \approx 0.242$, saddle point $(0.312, 0)$, lowest point $(-1.714, 0, -9.200)$
 27. Maxima $f(-1.267, 0) \approx 1.310$, $f(1.629, \pm 1.063) \approx 8.105$, saddle points $(-0.259, 0)$, $(1.526, 0)$, highest points $(1.629, \pm 1.063, 8.105)$
 29. Maximum $f(2, 0) = 9$, minimum $f(0, 3) = -14$
 31. Maximum $f(\pm 1, 1) = 7$, minimum $f(0, 0) = 4$
 33. Maximum $f(3, 0) = 83$, minimum $f(1, 1) = 0$
 35. Maximum $f(1, 0) = 2$, minimum $f(-1, 0) = -2$



39. $\sqrt{3}$ 41. $(2, 1, \sqrt{5}), (2, 1, -\sqrt{5})$ 43. $\frac{100}{3}, \frac{100}{3}, \frac{100}{3}$
 45. $8r^3/(3\sqrt{3})$ 47. $\frac{4}{3}$ 49. Cube, edge length $c/12$
 51. Square base of side 40 cm, height 20 cm 53. $L^3/(3\sqrt{3})$

EXERCISES 15.8 ■ PAGE 976

1. $\approx 59, 30$
 3. No maximum, minima $f(1, 1) = f(-1, -1) = 2$
 5. Maxima $f(\pm 2, 1) = 4$, minima $f(\pm 2, -1) = -4$
 7. Maximum $f(1, 3, 5) = 70$, minimum $f(-1, -3, -5) = -70$
 9. Maximum $2/\sqrt{3}$, minimum $-2/\sqrt{3}$
 11. Maximum $\sqrt{3}$, minimum 1
 13. Maximum $f(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}) = 2$, minimum $f(-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}) = -2$
 15. Maximum $f(1, \sqrt{2}, -\sqrt{2}) = 1 + 2\sqrt{2}$, minimum $f(1, -\sqrt{2}, \sqrt{2}) = 1 - 2\sqrt{2}$
 17. Maximum $\frac{3}{2}$, minimum $\frac{1}{2}$
 19. Maxima $f(\pm 1/\sqrt{2}, \mp 1/(2\sqrt{2})) = e^{1/4}$, minima $f(\pm 1/\sqrt{2}, \pm 1/(2\sqrt{2})) = e^{-1/4}$
 27–37. See Exercises 39–49 in Section 15.7.
 39. $L^3/(3\sqrt{3})$

41. Nearest $(\frac{1}{2}, \frac{1}{2}, \frac{1}{2})$, farthest $(-1, -1, 2)$
 43. Maximum ≈ 9.7938 , minimum ≈ -5.3506
 45. (a) c/n (b) When $x_1 = x_2 = \dots = x_n$

CHAPTER 15 REVIEW ■ PAGE 980

True-False Quiz

- I. True III. False V. False VII. True IX. False
 II. True

Exercises

1. $\{(x, y) | y > -x - 1\}$

 3.
 5.
 7.
 9. $\frac{2}{3}$
 11. (a) $\approx 3.5^\circ\text{C}/\text{m}$, $-3.0^\circ\text{C}/\text{m}$ (b) $\approx 0.35^\circ\text{C}/\text{m}$ by Equation 15.6.9 (Definition 15.6.2 gives $\approx 1.1^\circ\text{C}/\text{m}$.)
 (c) -0.25
 13. $f_x = 1/\sqrt{2x+y^2}$, $f_y = y/\sqrt{2x+y^2}$
 15. $g_u = \tan^{-1}v$, $g_v = u/(1+v^2)$
 17. $T_p = \ln(q+e^r)$, $T_q = p/(q+e^r)$, $T_r = pe^r/(q+e^r)$
 19. $f_{xx} = 24x$, $f_{xy} = -2y = f_{yx}$, $f_{yy} = -2x$
 21. $f_{xx} = k(k-1)x^{k-2}y^l z^m$, $f_{xy} = klx^{k-1}y^{l-1}z^m = f_{yx}$,
 $f_{xz} = kmx^{k-1}y^l z^{m-1} = f_{zx}$, $f_{yy} = l(l-1)x^ky^{l-2}z^m$,
 $f_{yz} = lmx^ky^{l-1}z^{m-1} = f_{zy}$, $f_{zz} = m(m-1)x^ky^l z^{m-2}$
 25. (a) $z = 8x + 4y + 1$ (b) $\frac{x-1}{8} = \frac{y+2}{4} = 1-z$
 27. (a) $2x - 2y - 3z = 3$ (b) $\frac{x-2}{4} = \frac{y+1}{-4} = \frac{z-1}{-6}$
 29. (a) $4x - y - 2z = 6$
 (b) $x = 3 + 8t$, $y = 4 - 2t$, $z = 1 - 4t$
 31. $(2, \frac{1}{2}, -1)$, $(-2, -\frac{1}{2}, 1)$
 33. $60x + \frac{24}{5}y + \frac{32}{5}z - 120; 38.656$
 35. $2xy^3(1+6p) + 3x^2y^2(pe^p + e^p) + 4z^3(p \cos p + \sin p)$
 37. $-47, 108$ 43. $ze^{x\sqrt{y}}(z\sqrt{y}, xz/(2\sqrt{y}), 2)$ 45. $\frac{43}{5}$
 47. $\sqrt{145}/2, \langle 4, \frac{9}{2} \rangle$ 49. $\approx \frac{5}{8}$ knot/mi

51. Minimum $f(-4, 1) = -11$
 53. Maximum $f(1, 1) = 1$; saddle points $(0, 0), (0, 3), (3, 0)$
 55. Maximum $f(1, 2) = 4$, minimum $f(2, 4) = -64$
 57. Maximum $f(-1, 0) = 2$, minima $f(1, \pm 1) = -3$, saddle points $(-1, \pm 1), (1, 0)$
 59. Maximum $f(\pm\sqrt{2}/3, 1/\sqrt{3}) = 2/(3\sqrt{3})$, minimum $f(\pm\sqrt{2}/3, -1/\sqrt{3}) = -2/(3\sqrt{3})$
 61. Maximum 1, minimum -1
 63. $(\pm 3^{-1/4}, 3^{-1/4}\sqrt{2}, \pm 3^{1/4}), (\pm 3^{-1/4}, -3^{-1/4}\sqrt{2}, \pm 3^{1/4})$
 65. $P(2 - \sqrt{3}), P(3 - \sqrt{3})/6, P(2\sqrt{3} - 3)/3$

PROBLEMS PLUS ■ PAGE 984

1. $L^2W^2, \frac{1}{4}L^2W^2$ 3. (a) $x = w/3$, base = $w/3$ (b) Yes
 7. $\sqrt{6}/2, 3\sqrt{2}/2$

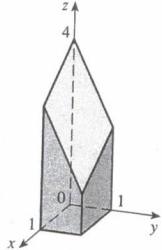
CHAPTER 16
EXERCISES 16.1 ■ PAGE 994

1. (a) 288 (b) 144
 3. (a) $\pi^2/2 \approx 4.935$ (b) 0
 5. (a) -6 (b) -3.5
 7. $U < V < L$
 9. (a) ≈ 248 (b) 15.5
 11. 60 13. 3
 15. 1.141606, 1.143191, 1.143535, 1.143617, 1.143637, 1.143642

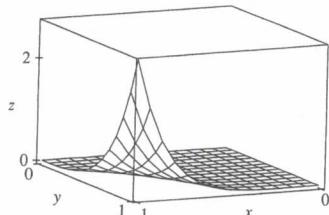
EXERCISES 16.2 ■ PAGE 1000

1. $500y^3, 3x^2$ 3. 10 5. 2 7. $261,632/45$ 9. $\frac{21}{2} \ln 2$
 11. 0 13. π 15. $\frac{21}{2}$ 17. $9 \ln 2$
 19. $\frac{1}{2}(\sqrt{3} - 1) - \frac{1}{12}\pi$ 21. $\frac{1}{2}(e^2 - 3)$

23.



25. 47.5 27. $\frac{166}{27}$ 29. 2 31. $\frac{64}{3}$
 33. $21e - 57$

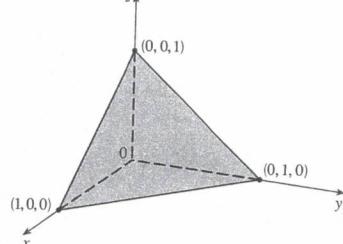

 35. $\frac{5}{6}$

37. Fubini's Theorem does not apply. The integrand has an infinite discontinuity at the origin.

EXERCISES 16.3 ■ PAGE 1008

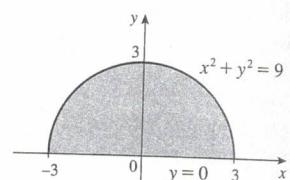
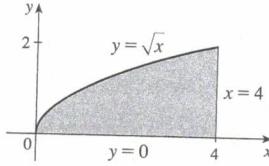
1. 32 3. $\frac{3}{10}$ 5. $e - 1$ 7. $\frac{4}{3}$ 9. π 11. $\frac{1}{2}e^{16} - \frac{17}{2}$
 13. $\frac{1}{2}(1 - \cos 1)$ 15. $\frac{147}{20}$ 17. 0 19. $\frac{7}{18}$ 21. $\frac{31}{8}$
 23. 6 25. $\frac{128}{15}$ 27. $\frac{1}{3}$ 29. 0, 1.213, 0.713 31. $\frac{64}{3}$

33.

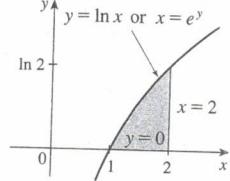


35. 13,984,735,616/14,549,535 37. $\pi/2$

39. $\int_0^2 \int_{y^2}^4 f(x, y) dx dy$ 41. $\int_{-3}^3 \int_0^{\sqrt{9-x^2}} f(x, y) dy dx$



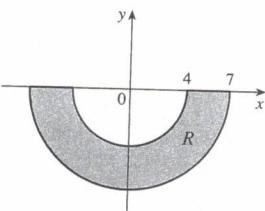
43. $\int_0^{\ln 2} \int_{e^y}^2 f(x, y) dx dy$



45. $\frac{1}{6}(e^9 - 1)$ 47. $\frac{1}{3} \ln 9$ 49. $\frac{1}{3}(2\sqrt{2} - 1)$ 51. 1
 53. $(\pi/16)e^{-1/16} \leq \iint_Q e^{-(x^2+y^2)^2} dA \leq \pi/16$ 55. $\frac{3}{4}$
 59. 8π 61. $2\pi/3$

EXERCISES 16.4 ■ PAGE 1014

1. $\int_0^{3\pi/2} \int_0^4 f(r \cos \theta, r \sin \theta) r dr d\theta$ 3. $\int_{-1}^1 \int_0^{(x+1)/2} f(x, y) dy dx$
 5.



7. 0 9. $\frac{1}{2}\pi \sin 9$ 11. $(\pi/2)(1 - e^{-4})$ 13. $\frac{3}{64}\pi^2$
 15. $\pi/12$ 17. $\frac{1}{8}(\pi - 2)$ 19. $\frac{16}{3}\pi$ 21. $\frac{4}{3}\pi$
 23. $\frac{4}{3}\pi a^3$ 25. $(2\pi/3)[1 - (1/\sqrt{2})]$
 27. $(8\pi/3)(64 - 24\sqrt{3})$
 29. $\frac{1}{2}\pi(1 - \cos 9)$, 31. $2\sqrt{2}/3$
 33. $1800\pi \text{ ft}^3$ 35. $\frac{15}{16}$ 37. (a) $\sqrt{\pi}/4$ (b) $\sqrt{\pi}/2$

EXERCISES 16.5 ■ PAGE 1024

1. $\frac{64}{3}$ C 3. $\left(\frac{4}{3}, 0\right)$ 5. $6, \left(\frac{3}{4}, \frac{3}{2}\right)$

7. $\frac{1}{4}(e^2 - 1), \left(\frac{e^2 + 1}{2(e^2 - 1)}, \frac{4(e^3 - 1)}{9(e^2 - 1)}\right)$

9. $L/4, (L/2, 16/(9\pi))$ 11. $(\frac{3}{8}, 3\pi/16)$ 13. $(0, 45/(14\pi))$

15. $(2a/5, 2a/5)$ if vertex is $(0, 0)$ and sides are along positive axes

17. $\frac{1}{16}(e^4 - 1), \frac{1}{8}(e^2 - 1), \frac{1}{16}(e^4 + 2e^2 - 3)$

19. $7ka^6/180, 7ka^6/180, 7ka^6/90$ if vertex is $(0, 0)$ and sides are along positive axes

21. $m = \pi^2/8, (\bar{x}, \bar{y}) = \left(\frac{2\pi}{3} - \frac{1}{\pi}, \frac{16}{9\pi}\right), I_x = 3\pi^2/64,$

$I_y = \frac{1}{16}(\pi^4 - 3\pi^2), I_0 = \pi^4/16 - 9\pi^2/64$

23. $p\bar{b}h^3/3, p\bar{b}^3h/3; b/\sqrt{3}, h/\sqrt{3}$

25. $pa^4\pi/16, pa^4\pi/16; a/2, a/2$

27. (a) $\frac{1}{2}$ (b) 0.375 (c) $\frac{5}{48} \approx 0.1042$

29. (b) (i) $e^{-0.2} \approx 0.8187$

(ii) $1 + e^{-1.8} - e^{-0.8} - e^{-1} \approx 0.3481$ (c) 2, 5

31. (a) ≈ 0.500 (b) ≈ 0.632

33. (a) $\iint_D (k/20)[20 - \sqrt{(x - x_0)^2 + (y - y_0)^2}] dA$, where D is the disk with radius 10 mi centered at the center of the city(b) $200\pi k/3 \approx 209k, 200(\pi/2 - \frac{8}{9})k \approx 136k$, on the edge

EXERCISES 16.6 ■ PAGE 1034

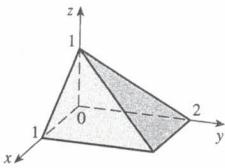
1. $\frac{27}{4}$ 3. 1 5. $\frac{1}{3}(e^3 - 1)$ 7. $-\frac{1}{3}$ 9. 4 11. $\frac{65}{28}$

13. $8/(3e)$ 15. $\frac{1}{60}$ 17. $16\pi/3$ 19. $\frac{16}{3}$ 21. 36π

23. (a) $\int_0^1 \int_0^x \int_0^{\sqrt{1-y^2}} dz dy dx$ (b) $\frac{1}{4}\pi - \frac{1}{3}$

25. 60.533

27.



$$\begin{aligned} 29. & \int_{-2}^2 \int_0^{4-x^2} \int_{-\sqrt{4-x^2-y^2}/2}^{\sqrt{4-x^2-y^2}/2} f(x, y, z) dz dy dx \\ &= \int_0^4 \int_{-\sqrt{4-y}}^{\sqrt{4-y}} \int_{-\sqrt{4-x^2-y^2}/2}^{\sqrt{4-x^2-y^2}/2} f(x, y, z) dz dy dx \\ &= \int_{-1}^1 \int_0^{4-4z^2} \int_{-\sqrt{4-y-4z^2}}^{\sqrt{4-y-4z^2}} f(x, y, z) dx dy dz \\ &= \int_0^4 \int_{-\sqrt{4-y}/2}^{\sqrt{4-y}/2} \int_{-\sqrt{4-y-4z^2}}^{\sqrt{4-y-4z^2}} f(x, y, z) dx dy dz \\ &= \int_{-2}^2 \int_{-\sqrt{4-x^2}/2}^{\sqrt{4-x^2}/2} \int_0^{4-x^2-4z^2} f(x, y, z) dy dx dz \\ &= \int_{-1}^1 \int_{-\sqrt{4-4z^2}}^{\sqrt{4-4z^2}} \int_0^{4-x^2-4z^2} f(x, y, z) dy dx dz \end{aligned}$$

31. $\int_{-2}^2 \int_x^2 \int_0^{2-y/2} f(x, y, z) dz dy dx$

= $\int_0^4 \int_{-\sqrt{y}}^{\sqrt{y}} \int_0^{2-y/2} f(x, y, z) dz dx dy$

= $\int_0^2 \int_0^{4-2z} \int_{-\sqrt{y}}^{\sqrt{y}} f(x, y, z) dx dy dz$

= $\int_0^4 \int_0^{2-y/2} \int_{-\sqrt{y}}^{\sqrt{y}} f(x, y, z) dx dz dy$

= $\int_{-2}^2 \int_0^{2-x^2/2} \int_{x^2}^{4-2z} f(x, y, z) dy dz dx$

= $\int_0^2 \int_{-\sqrt{4-2z}}^{\sqrt{4-2z}} \int_{x^2}^{4-2z} f(x, y, z) dy dx dz$

33. $\int_0^1 \int_{\sqrt{x}}^1 \int_0^{1-y} f(x, y, z) dz dy dx$

= $\int_0^1 \int_0^{1-z} \int_0^{1-y} f(x, y, z) dz dx dy$

= $\int_0^1 \int_0^{1-z} \int_0^y f(x, y, z) dx dy dz$

= $\int_0^1 \int_0^{1-y} \int_0^y f(x, y, z) dx dz dy$

= $\int_0^1 \int_0^{1-y} \int_{\sqrt{x}}^{1-z} f(x, y, z) dy dz dx$

= $\int_0^1 \int_0^{(1-z)^2} \int_{\sqrt{x}}^{1-z} f(x, y, z) dy dx dz$

35. $\int_0^1 \int_0^1 \int_0^y f(x, y, z) dz dx dy = \int_0^1 \int_0^x \int_0^y f(x, y, z) dz dy dx$

= $\int_0^1 \int_z^1 \int_0^y f(x, y, z) dx dy dz = \int_0^1 \int_0^y \int_z^1 f(x, y, z) dx dz dy$

= $\int_0^1 \int_0^y \int_z^x f(x, y, z) dy dz dx = \int_0^1 \int_z^1 \int_z^x f(x, y, z) dy dx dz$

37. $\frac{79}{30}, \left(\frac{358}{553}, \frac{33}{79}, \frac{571}{553}\right)$ 39. $a^5, (7a/12, 7a/12, 7a/12)$

41. $I_x = I_y = I_z = \frac{2}{3}kL^5$ 43. $\frac{1}{2}\pi kha^4$

45. (a) $m = \int_{-3}^3 \int_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}} \int_1^{5-y} \sqrt{x^2 + y^2} dz dy dx$

(b) $(\bar{x}, \bar{y}, \bar{z})$, where

$\bar{x} = (1/m) \int_{-3}^3 \int_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}} \int_1^{5-y} x \sqrt{x^2 + y^2} dz dy dx$

$\bar{y} = (1/m) \int_{-3}^3 \int_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}} \int_1^{5-y} y \sqrt{x^2 + y^2} dz dy dx$

$\bar{z} = (1/m) \int_{-3}^3 \int_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}} \int_1^{5-y} z \sqrt{x^2 + y^2} dz dy dx$

(c) $\int_{-3}^3 \int_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}} \int_1^{5-y} (x^2 + y^2)^{3/2} dz dy dx$

47. (a) $\frac{3}{32}\pi + \frac{11}{24}$

(b) $(\bar{x}, \bar{y}, \bar{z}) = \left(\frac{28}{9\pi + 44}, \frac{30\pi + 128}{45\pi + 220}, \frac{45\pi + 208}{135\pi + 660}\right)$

(c) $\frac{1}{240}(68 + 15\pi)$

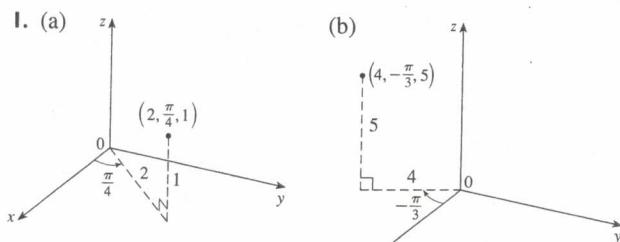
49. (a) $\frac{1}{8}$ (b) $\frac{1}{64}$ (c) $\frac{1}{5760}$

51. $L^3/8$

53. The region bounded by the ellipsoid $x^2 + 2y^2 + 3z^2 = 1$

EXERCISES 16.7 ■ PAGE 1040

1. (a)



$(\sqrt{2}, \sqrt{2}, 1)$

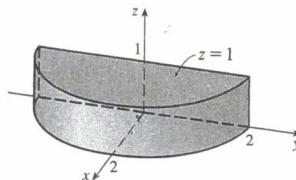
$(2, -2\sqrt{3}, 5)$

3. (a) $(\sqrt{2}, 7\pi/4, 4)$ (b) $(2, 4\pi/3, 2)$

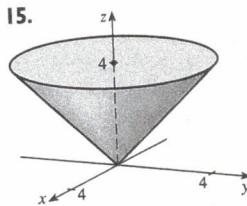
5. Vertical half-plane through the z -axis

9. (a) $z = r^2$ (b) $r = 2 \sin \theta$

11.



13. Cylindrical coordinates: $6 \leq r \leq 7, 0 \leq \theta \leq 2\pi, 0 \leq z \leq 20$



$$64\pi/3$$

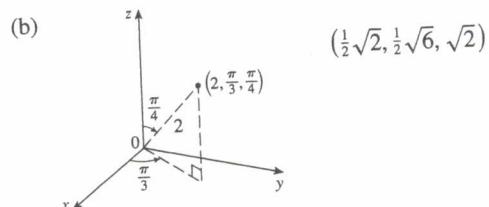
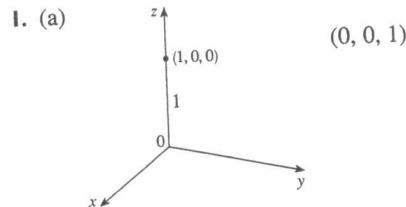
15. 384π 19. $\pi(e^6 - e - 5)$ 21. $2\pi/5$

23. (a) 162π (b) $(0, 0, 15)$

25. $\pi Ka^2/8, (0, 0, 2a/3)$ 27. 0

29. (a) $\iiint_C h(P)g(P) dV$, where C is the cone
(b) $\approx 3.1 \times 10^{19}$ ft-lb

EXERCISES 16.8 ■ PAGE 1046

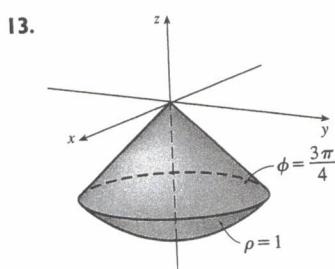
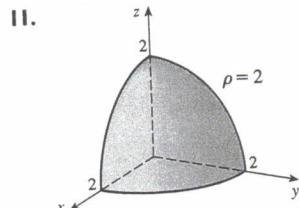


3. (a) $(4, \pi/3, \pi/6)$ (b) $(\sqrt{2}, 3\pi/2, 3\pi/4)$

5. Half-cone

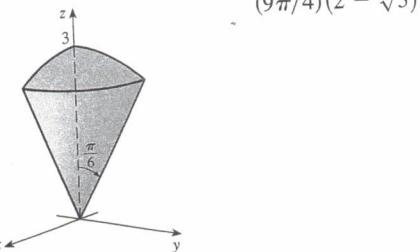
7. Sphere, radius $\frac{1}{2}$, center $(0, \frac{1}{2}, 0)$

9. (a) $\cos^2\phi = \sin^2\phi$ (b) $\rho^2(\sin^2\phi \cos^2\theta + \cos^2\phi) = 9$



15. $0 \leq \phi \leq \pi/4, 0 \leq \rho \leq \cos \phi$

17.



$$(9\pi/4)(2 - \sqrt{3})$$

19. $\int_0^{\pi/2} \int_0^3 \int_0^2 f(r \cos \theta, r \sin \theta, z) r dz dr d\theta$

21. $312,500\pi/7$ 23. $15\pi/16$ 25. $1562\pi/15$

27. $(\sqrt{3} - 1)\pi a^3/3$

29. (a) 10π (b) $(0, 0, 2.1)$

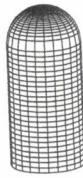
31. $(0, \frac{525}{296}, 0)$

33. (a) $(0, 0, \frac{3}{8}a)$ (b) $4K\pi a^5/15$

35. $(2\pi/3)[1 - (1/\sqrt{2})], (0, 0, 3/[8(2 - \sqrt{2})])$

37. $5\pi/6$ 39. $(4\sqrt{2} - 5)/15$

41. $136\pi/99$



EXERCISES 16.9 ■ PAGE 1056

1. 16 3. $\sin^2\theta - \cos^2\theta$ 5. 0

7. The parallelogram with vertices $(0, 0), (6, 3), (12, 1), (6, -2)$

9. The region bounded by the line $y = 1$, the y -axis, and $y = \sqrt{x}$

11. -3 13. 6π 15. $2 \ln 3$

17. (a) $\frac{4}{3}\pi abc$ (b) $1.083 \times 10^{12} \text{ km}^3$

19. $\frac{8}{5} \ln 8$ 21. $\frac{3}{2} \sin 1$ 23. $e - e^{-1}$

CHAPTER 16 REVIEW ■ PAGE 1057

True-False Quiz

1. True 3. True 5. True 7. False

Exercises

1. ≈ 64.0 3. $4e^2 - 4e + 3$ 5. $\frac{1}{2} \sin 1$ 7. $\frac{2}{3}$

9. $\int_0^\pi \int_2^4 f(r \cos \theta, r \sin \theta) r dr d\theta$

11. The region inside the loop of the four-leaved rose $r = \sin 2\theta$ in the first quadrant

13. $\frac{1}{2} \sin 1$ 15. $\frac{1}{2} e^6 - \frac{7}{2}$ 17. $\frac{1}{4} \ln 2$ 19. 8

21. $81\pi/5$ 23. 40.5 25. $\pi/96$ 27. $\frac{64}{15}$ 29. 176

31. $\frac{2}{3}$ 33. $2ma^3/9$

35. (a) $\frac{1}{4}$ (b) $(\frac{1}{3}, \frac{8}{15})$

(c) $I_x = \frac{1}{12}, I_y = \frac{1}{24}; \bar{y} = 1/\sqrt{3}, \bar{x} = 1/\sqrt{6}$

37. $(0, 0, h/4)$

39. 97.2 41. 0.0512

43. (a) $\frac{1}{15}$ (b) $\frac{1}{3}$ (c) $\frac{1}{45}$

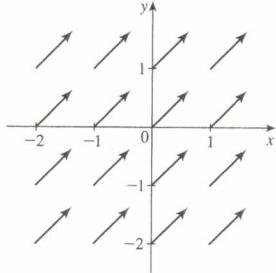
45. $\int_0^1 \int_0^{1-z} \int_{-\sqrt{y}}^{\sqrt{y}} f(x, y, z) dx dy dz$ 47. $-\ln 2$ 49. 0

PROBLEMS PLUS ■ PAGE 1060

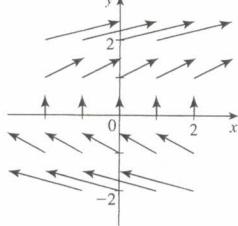
1. 30 3. $\frac{1}{2} \sin 1$ 7. (b) 0.90

CHAPTER 17**EXERCISES 17.1 ■ PAGE 1068**

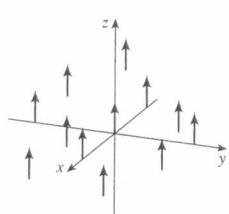
1.



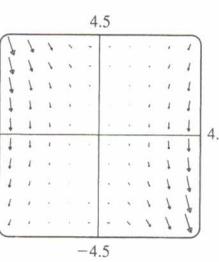
3.



7.



II.



13. I

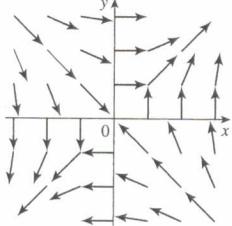
15. IV

17. III

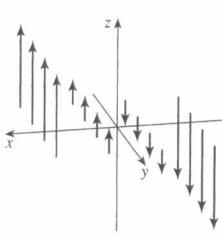
The line $y = 2x$

19.

5.



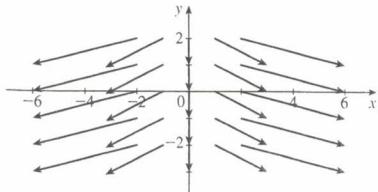
9.



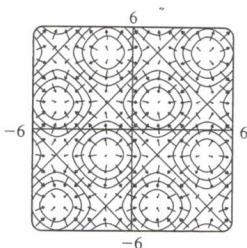
21. $\nabla f(x, y) = (xy + 1)e^{xy} \mathbf{i} + x^2 e^{xy} \mathbf{j}$

23. $\nabla f(x, y, z) = \frac{x}{\sqrt{x^2 + y^2 + z^2}} \mathbf{i} + \frac{y}{\sqrt{x^2 + y^2 + z^2}} \mathbf{j} + \frac{z}{\sqrt{x^2 + y^2 + z^2}} \mathbf{k}$

25. $\nabla f(x, y) = 2x \mathbf{i} - \mathbf{j}$



27.

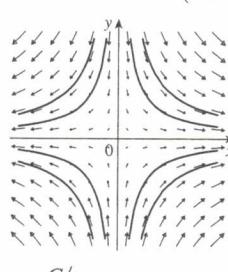


29. III

31. II

33. (2.04, 1.03)

35. (a)

(b) $y = 1/x, x > 0$ **EXERCISES 17.2 ■ PAGE 1079**

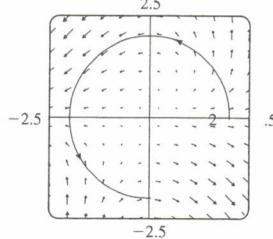
1. $\frac{1}{54}(145^{3/2} - 1)$ 3. 1638.4 5. $\frac{243}{8}$ 7. $\frac{17}{3}$ 9. $\sqrt{5}\pi$

11. $\frac{1}{12}\sqrt{14}(e^6 - 1)$ 13. $\frac{1}{5}$ 15. $\frac{97}{3}$

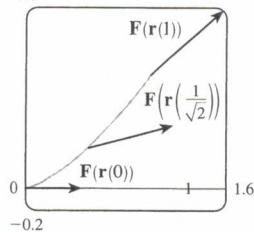
17. (a) Positive (b) Negative

19. 45 21. $\frac{6}{5} - \cos 1 - \sin 1$ 23. 1.9633 25. 15.0074

27. $3\pi + \frac{2}{3}$



29. (a) $\frac{11}{8} - 1/e$ (b) 1.6



31. $\frac{172,704}{5,632,705}\sqrt{2}(1 - e^{-14\pi})$ 33. $2\pi k, (4/\pi, 0)$

35. (a) $\bar{x} = (1/m) \int_C x\rho(x, y, z) ds$,

$\bar{y} = (1/m) \int_C y\rho(x, y, z) ds$,

$\bar{z} = (1/m) \int_C z\rho(x, y, z) ds$, where $m = \int_C \rho(x, y, z) ds$

(b) $(0, 0, 3\pi)$

37. $I_x = k\left(\frac{1}{2}\pi - \frac{4}{3}\right)$, $I_y = k\left(\frac{1}{2}\pi - \frac{2}{3}\right)$

39. $2\pi^2$ 41. 26 43. 1.67×10^4 ft-lb 45. (b) Yes

47. ≈ 22 J

EXERCISES 17.3 ■ PAGE 1089

1. 40 3. $f(x, y) = x^2 - 3xy + 2y^2 - 8y + K$
 5. $f(x, y) = e^x \sin y + K$ 7. $f(x, y) = ye^x + x \sin y + K$
 9. $f(x, y) = x \ln y + x^2 y^3 + K$
 11. (b) 16 13. (a) $f(x, y) = \frac{1}{2}x^2 y^2$ (b) 2
 15. (a) $f(x, y, z) = xyz + z^2$ (b) 77
 17. (a) $f(x, y, z) = xy^2 \cos z$ (b) 0
 19. 2 21. 30 23. No 25. Conservative
 29. (a) Yes (b) Yes (c) Yes
 31. (a) Yes (b) Yes (c) No

EXERCISES 17.4 ■ PAGE 1096

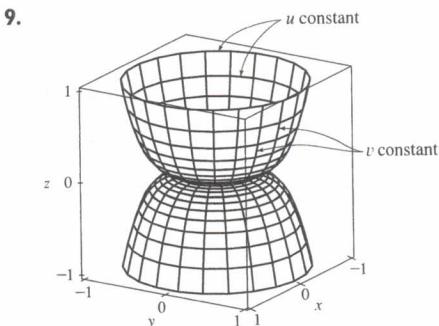
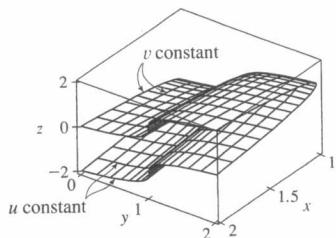
1. 8π 3. $\frac{2}{3}$ 5. 12 7. $\frac{1}{3}$ 9. -24π 11. $\frac{4}{3} - 2\pi$
 13. $\frac{625}{2}\pi$ 15. $-8e + 48e^{-1}$ 17. $-\frac{1}{12}$ 19. 3π 21. (c) $\frac{9}{2}$
 23. $(4a/3\pi, 4a/3\pi)$ if the region is the portion of the disk
 $x^2 + y^2 = a^2$ in the first quadrant

EXERCISES 17.5 ■ PAGE 1104

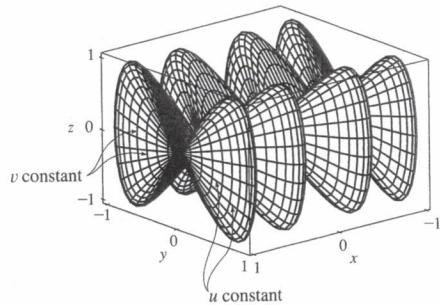
1. (a) $-x^2 \mathbf{i} + 3xy \mathbf{j} - xz \mathbf{k}$ (b) yz
 3. (a) $(x - y)\mathbf{i} - y\mathbf{j} + \mathbf{k}$ (b) $z - 1/(2\sqrt{z})$
 5. (a) $\mathbf{0}$ (b) $2/\sqrt{x^2 + y^2 + z^2}$
 7. (a) $\langle 1/y, -1/x, 1/x \rangle$ (b) $1/x + 1/y + 1/z$
 9. (a) Negative (b) $\operatorname{curl} \mathbf{F} = \mathbf{0}$
 11. (a) Zero (b) $\operatorname{curl} \mathbf{F}$ points in the negative z -direction
 13. $f(x, y, z) = xy^2 z^3 + K$ 15. $f(x, y, z) = x^2 y + y^2 z + K$
 17. Not conservative 19. No

EXERCISES 17.6 ■ PAGE 1114

1. P: no; Q: yes
 3. Plane through $(0, 3, 1)$ containing vectors $\langle 1, 0, 4 \rangle, \langle 1, -1, 5 \rangle$
 5. Hyperbolic paraboloid
 7.



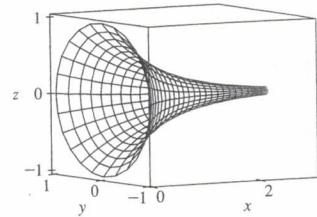
11.



13. IV 15. II 17. III

19. $x = 1 + u + v, y = 2 + u - v, z = -3 - u + v$
 21. $x = x, z = z, y = \sqrt{1 - x^2 + z^2}$
 23. $x = 2 \sin \phi \cos \theta, y = 2 \sin \phi \sin \theta,$
 $z = 2 \cos \phi, 0 \leq \phi \leq \pi/4, 0 \leq \theta \leq 2\pi$
 [or $x = x, y = y, z = \sqrt{4 - x^2 - y^2}, x^2 + y^2 \leq 2$]
 25. $x = x, y = 4 \cos \theta, z = 4 \sin \theta, 0 \leq x \leq 5, 0 \leq \theta \leq 2\pi$

29. $x = x, y = e^{-x} \cos \theta,$
 $z = e^{-x} \sin \theta, 0 \leq x \leq 3,$
 $0 \leq \theta \leq 2\pi$



31. (a) Direction reverses (b) Number of coils doubles
 33. $3x - y + 3z = 3$ 35. $-x + 2z = 1$ 37. $3\sqrt{14}$

39. $\frac{4}{15}(3^{5/2} - 2^{7/2} + 1)$ 41. $(2\pi/3)(2\sqrt{2} - 1)$

43. $(\pi/6)(17\sqrt{17} - 5\sqrt{5})$

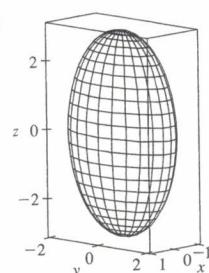
45. $\frac{1}{2}\sqrt{21} + \frac{17}{4}[\ln(2 + \sqrt{21}) - \ln\sqrt{17}]$ 47. 4

49. 13.9783

51. (a) 24.2055 (b) 24.2476

53. $\frac{45}{8}\sqrt{14} + \frac{15}{16}\ln[(11\sqrt{5} + 3\sqrt{70})/(3\sqrt{5} + \sqrt{70})]$

55. (b)



- (c) $\int_0^{2\pi} \int_0^\pi \sqrt{36 \sin^4 u \cos^2 v + 9 \sin^4 u \sin^2 v + 4 \cos^2 u \sin^2 u} du dv$
 57. 4π 59. $2a^2(\pi - 2)$

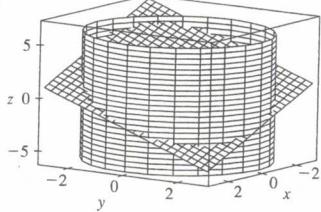
EXERCISES 17.7 ■ PAGE 1127

1. 49.09 3. 900π 5. $171\sqrt{14}$ 7. $\sqrt{3}/24$
 9. $5\sqrt{5}/48 + 1/240$ 11. $364\sqrt{2}\pi/3$
 13. $(\pi/60)(391\sqrt{17} + 1)$ 15. 16π 17. 12

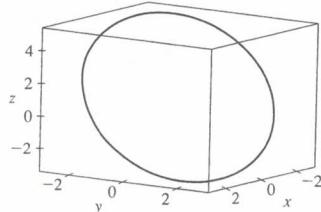
19. $\frac{713}{180}$ 21. $-\frac{1}{6}$ 23. $-\frac{4}{3}\pi$ 25. 0 27. 48
 29. $2\pi + \frac{8}{3}$ 31. 0.1642 33. 3.4895
 35. $\iint_S \mathbf{F} \cdot d\mathbf{S} = \iint_D [P(\partial h/\partial x) - Q + R(\partial h/\partial z)] dA$,
 where D = projection of S on xz -plane
 37. $(0, 0, a/2)$
 39. (a) $I_z = \iint_S (x^2 + y^2)\rho(x, y, z) dS$ (b) $4329\sqrt{2}\pi/5$
 41. 0 kg/s 43. $\frac{8}{3}\pi a^3 \epsilon_0$ 45. 1248π

EXERCISES 17.8 ■ PAGE 1133

3. 0 5. 0 7. -1 9. 80π
 11. (a) $81\pi/2$ (b)



(c) $x = 3 \cos t, y = 3 \sin t, z = 1 - 3(\cos t + \sin t), 0 \leq t \leq 2\pi$



17. 3

EXERCISES 17.9 ■ PAGE 1139

5. 2 7. $9\pi/2$
 9. 0 11. $32\pi/3$ 13. 0
 15. $341\sqrt{2}/60 + \frac{81}{20} \arcsin(\sqrt{3}/3)$ 17. $13\pi/20$
 19. Negative at P_1 , positive at P_2
 21. $\operatorname{div} \mathbf{F} > 0$ in quadrants I, II; $\operatorname{div} \mathbf{F} < 0$ in quadrants III, IV

CHAPTER 17 REVIEW ■ PAGE 1142

True-False Quiz

1. False 3. True 5. False 7. True

Exercises

1. (a) Negative (b) Positive 3. $6\sqrt{10}$ 5. $\frac{4}{15}$
 7. $\frac{110}{3}$ 9. $\frac{11}{12} - 4/e$ 11. $f(x, y) = e^y + xe^{xy}$ 13. 0
 17. -8π 25. $\frac{1}{6}(27 - 5\sqrt{5})$
 27. $(\pi/60)(391\sqrt{17} + 1)$ 29. $-64\pi/3$
 33. $-\frac{1}{2}$ 37. -4 39. 21

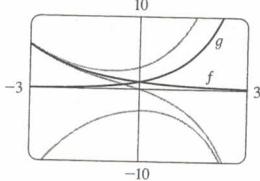
APPENDIX H ANSWERS TO ODD-NUMBERED EXERCISES

CHAPTER 18

EXERCISES 18.1 ■ PAGE 1153

1. $y = c_1 e^{3x} + c_2 e^{-2x}$ 3. $y = c_1 \cos 4x + c_2 \sin 4x$
 5. $y = c_1 e^{2x/3} + c_2 x e^{2x/3}$ 7. $y = c_1 + c_2 e^{x/2}$
 9. $y = e^{2x}(c_1 \cos 3x + c_2 \sin 3x)$
 11. $y = c_1 e^{(\sqrt{3}-1)x/2} + c_2 e^{-(\sqrt{3}+1)x/2}$
 13. $P = e^{-t}[c_1 \cos(\frac{1}{10}t) + c_2 \sin(\frac{1}{10}t)]$

15.



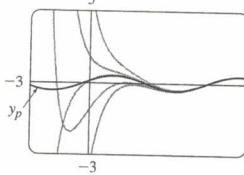
All solutions approach either 0 or $\pm\infty$ as $x \rightarrow \pm\infty$.

17. $y = 2e^{-3x/2} + e^{-x}$ 19. $y = e^{x/2} - 2xe^{x/2}$
 21. $y = 3 \cos 4x - \sin 4x$ 23. $y = e^{-x}(2 \cos x + 3 \sin x)$
 25. $y = 3 \cos(\frac{1}{2}x) - 4 \sin(\frac{1}{2}x)$ 27. $y = \frac{e^{x+3}}{e^3 - 1} + \frac{e^{2x}}{1 - e^3}$
 29. No solution
 31. $y = e^{-2x}(2 \cos 3x - e^\pi \sin 3x)$
 33. (b) $\lambda = n^2\pi^2/L^2$, n a positive integer; $y = C \sin(n\pi x/L)$

EXERCISES 18.2 ■ PAGE 1160

1. $y = c_1 e^{-2x} + c_2 e^{-x} + \frac{1}{2}x^2 - \frac{3}{2}x + \frac{7}{4}$
 3. $y = c_1 + c_2 e^{2x} + \frac{1}{40} \cos 4x - \frac{1}{20} \sin 4x$
 5. $y = e^{2x}(c_1 \cos x + c_2 \sin x) + \frac{1}{10}e^{-x}$
 7. $y = \frac{3}{2} \cos x + \frac{11}{2} \sin x + \frac{1}{2}e^x + x^3 - 6x$
 9. $y = e^x(\frac{1}{2}x^2 - x + 2)$

11.



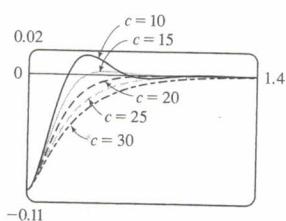
The solutions are all asymptotic to $y_p = \frac{1}{10} \cos x + \frac{3}{10} \sin x$ as $x \rightarrow \infty$. Except for y_p , all solutions approach either ∞ or $-\infty$ as $x \rightarrow -\infty$.

13. $y_p = Ae^{2x} + (Bx^2 + Cx + D) \cos x + (Ex^2 + Fx + G) \sin x$
 15. $y_p = Ax + (Bx + C)e^{9x}$
 17. $y_p = xe^{-x}[(Ax^2 + Bx + C) \cos 3x + (Dx^2 + Ex + F) \sin 3x]$
 19. $y = c_1 \cos(\frac{1}{2}x) + c_2 \sin(\frac{1}{2}x) - \frac{1}{3} \cos x$
 21. $y = c_1 e^x + c_2 x e^x + e^{2x}$
 23. $y = c_1 \sin x + c_2 \cos x + \sin x \ln(\sec x + \tan x) - 1$
 25. $y = [c_1 + \ln(1 + e^{-x})]e^x + [c_2 - e^{-x} + \ln(1 + e^{-x})]e^{2x}$
 27. $y = e^x[c_1 + c_2 x - \frac{1}{2} \ln(1 + x^2) + x \tan^{-1} x]$

EXERCISES 18.3 ■ PAGE 1168

1. $x = 0.35 \cos(2\sqrt{5}t)$ 3. $x = -\frac{1}{5}e^{-6t} + \frac{6}{5}e^{-t}$ 5. $\frac{49}{12} \text{ kg}$

7.



13. $Q(t) = (-e^{-10t}/250)(6 \cos 20t + 3 \sin 20t) + \frac{3}{125}$,
 $I(t) = \frac{3}{5}e^{-10t} \sin 20t$
15. $Q(t) = e^{-10t} \left[\frac{3}{250} \cos 20t - \frac{3}{500} \sin 20t \right] - \frac{3}{250} \cos 10t + \frac{3}{125} \sin 10t$

EXERCISES 18.4 ■ PAGE 1173

1. $c_0 \sum_{n=0}^{\infty} \frac{x^n}{n!} = c_0 e^x$ 3. $c_0 \sum_{n=0}^{\infty} \frac{x^{3n}}{3^n n!} = c_0 e^{x^3/3}$
5. $c_0 \sum_{n=0}^{\infty} \frac{(-1)^n}{2^n n!} x^{2n} + c_1 \sum_{n=0}^{\infty} \frac{(-2)^n n!}{(2n+1)!} x^{2n+1}$
7. $c_0 + c_1 \sum_{n=1}^{\infty} \frac{x^n}{n} = c_0 - c_1 \ln(1-x)$ for $|x| < 1$
9. $\sum_{n=0}^{\infty} \frac{x^{2n}}{2^n n!} = e^{x^2/2}$
11. $x + \sum_{n=1}^{\infty} \frac{(-1)^n 2^2 5^2 \cdots (3n-1)^2}{(3n+1)!} x^{3n+1}$

CHAPTER 18 REVIEW ■ PAGE 1174

True-False Quiz

1. True 3. True

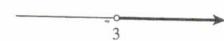
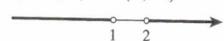
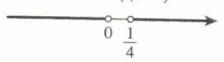
Exercises

1. $y = c_1 e^{5x} + c_2 e^{-3x}$ 3. $y = c_1 \cos(\sqrt{3}x) + c_2 \sin(\sqrt{3}x)$
5. $y = e^{2x}(c_1 \cos x + c_2 \sin x + 1)$
7. $y = c_1 e^x + c_2 x e^x - \frac{1}{2} \cos x - \frac{1}{2}(x+1) \sin x$
9. $y = c_1 e^{3x} + c_2 e^{-2x} - \frac{1}{6} - \frac{1}{5} x e^{-2x}$
11. $y = 5 - 2e^{-6(x-1)}$ 13. $y = (e^{4x} - e^x)/3$
15. $\sum_{n=0}^{\infty} \frac{(-2)^n n!}{(2n+1)!} x^{2n+1}$
17. $Q(t) = -0.02e^{-10t}(\cos 10t + \sin 10t) + 0.03$
19. (c) $2\pi/k \approx 85$ min (d) $\approx 17,600$ mi/h

APPENDICES

EXERCISES A ■ PAGE A9

1. 18 3. π 5. $5 - \sqrt{5}$ 7. $2 - x$
9. $|x+1| = \begin{cases} x+1 & \text{for } x \geq -1 \\ -x-1 & \text{for } x < -1 \end{cases}$ 11. $x^2 + 1$
13. $(-2, \infty)$ 15. $[-1, \infty)$

17. $(3, \infty)$ 21. $(0, 1]$ 25. $(-\infty, 1) \cup (2, \infty)$ 29. $(-\infty, \infty)$ 33. $(-\infty, 1]$ 37. $(-\infty, 0) \cup (\frac{1}{4}, \infty)$ 39. $10 \leq C \leq 35$ 41. (a) $T = 20 - 10h$, $0 \leq h \leq 12$ (b) $-30^\circ\text{C} \leq T \leq 20^\circ\text{C}$ 43. $\pm \frac{3}{2}$ 45. $2, -\frac{4}{3}$ 47. $(-3, 3)$ 49. $(3, 5)$ 51. $(-\infty, -7] \cup [-3, \infty)$ 53. $[1.3, 1.7]$ 55. $[-4, -1] \cup [1, 4]$ 57. $x \geq (a+b)c/(ab)$ 59. $x > (c-b)/a$

EXERCISES B ■ PAGE A15

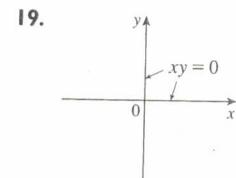
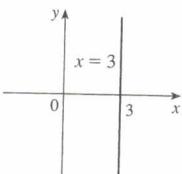
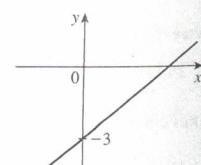
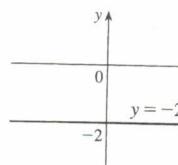
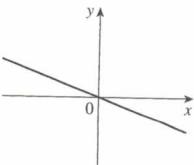
1. 5

3. $\sqrt{74}$ 5. $2\sqrt{37}$

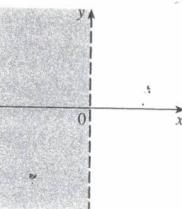
7. 2

9. $-\frac{9}{2}$

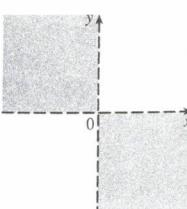
17.

21. $y = 6x - 15$ 23. $2x - 3y + 19 = 0$ 25. $5x + y = 11$ 27. $y = 3x - 2$ 29. $y = 3x - 3$ 31. $y = 5$ 33. $x + 2y + 11 = 0$ 35. $5x - 2y + 1 = 0$ 37. $m = -\frac{1}{3}$, $b = 0$ 39. $m = 0$, $b = -2$ 41. $m = \frac{3}{4}$, $b = -3$ 

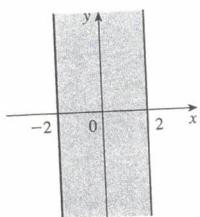
43.



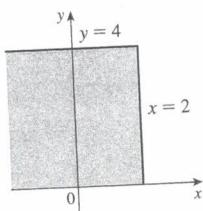
45.



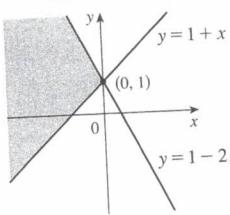
47.



49.



51.



53. $(0, -4)$

55. (a) $(4, 9)$ (b) $(3.5, -3)$

59. $y = x - 3$

61. (b) $4x - 3y - 24 = 0$

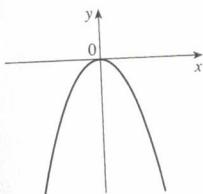
57. $(1, -2)$

EXERCISES C ■ PAGE A23

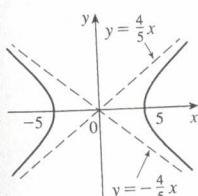
1. $(x - 3)^2 + (y + 1)^2 = 25$

5. $(2, -5), 4$

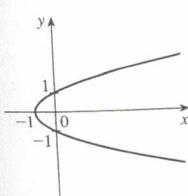
11. Parabola



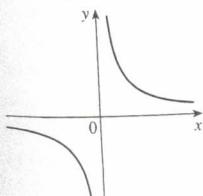
15. Hyperbola



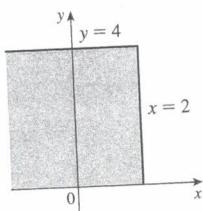
19. Parabola



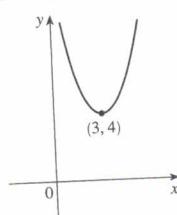
23. Hyperbola



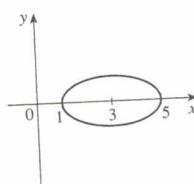
49.



27. Parabola



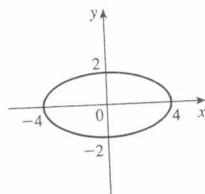
31. Ellipse



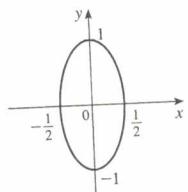
57. $(1, -2)$

3. $x^2 + y^2 = 65$
9. $(\frac{1}{4}, -\frac{1}{4}), \sqrt{10}/4$

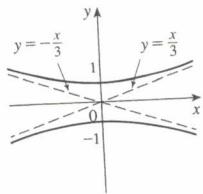
13. Ellipse



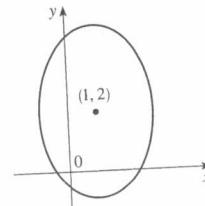
17. Ellipse



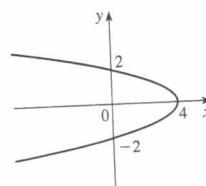
21. Hyperbola



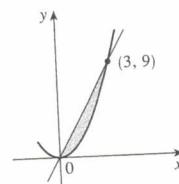
25. Ellipse



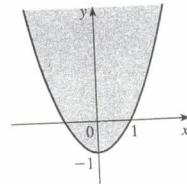
29. Parabola



33.



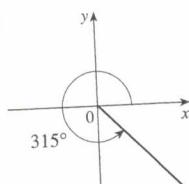
39.



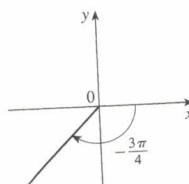
EXERCISES D ■ PAGE A32

1. $7\pi/6$ 3. $\pi/20$ 5. 5π 7. 720° 9. 75°
11. -67.5° 13. 3π cm 15. $\frac{2}{3}\text{ rad} = (120/\pi)^\circ$

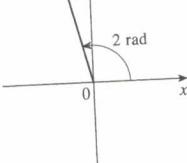
17.



19.



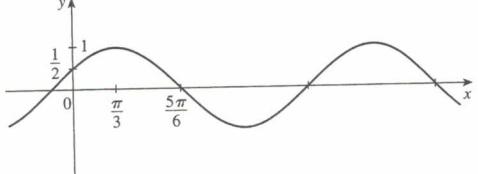
21.



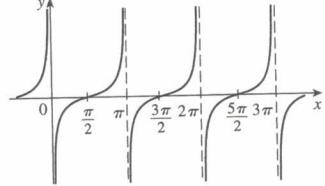
23. $\sin(3\pi/4) = 1/\sqrt{2}$, $\cos(3\pi/4) = -1/\sqrt{2}$, $\tan(3\pi/4) = -1$,
 $\csc(3\pi/4) = \sqrt{2}$, $\sec(3\pi/4) = -\sqrt{2}$, $\cot(3\pi/4) = -1$
25. $\sin(9\pi/2) = 1$, $\cos(9\pi/2) = 0$, $\csc(9\pi/2) = 1$, $\sec(9\pi/2) = 0$,
 $\tan(9\pi/2)$ and $\cot(9\pi/2)$ undefined
27. $\sin(5\pi/6) = \frac{1}{2}$, $\cos(5\pi/6) = -\sqrt{3}/2$, $\tan(5\pi/6) = -1/\sqrt{3}$,
 $\csc(5\pi/6) = 2$, $\sec(5\pi/6) = -2/\sqrt{3}$, $\cot(5\pi/6) = -\sqrt{3}$
29. $\cos \theta = \frac{4}{5}$, $\tan \theta = \frac{3}{4}$, $\csc \theta = \frac{5}{3}$, $\sec \theta = \frac{5}{4}$, $\cot \theta = \frac{4}{3}$
31. $\sin \phi = \frac{\sqrt{5}}{3}$, $\cos \phi = -\frac{2}{3}$, $\tan \phi = -\sqrt{5}/2$, $\csc \phi = 3/\sqrt{5}$,
 $\cot \phi = -2/\sqrt{5}$

33. $\sin \beta = -1/\sqrt{10}$, $\cos \beta = -3/\sqrt{10}$, $\tan \beta = \frac{1}{3}$,
 $\csc \beta = -\sqrt{10}$, $\sec \beta = -\sqrt{10}/3$
35. 5.73576 cm 37. 24.62147 cm 39. $\frac{1}{15}(4 + 6\sqrt{2})$
61. $\frac{1}{15}(3 + 8\sqrt{2})$ 63. $\frac{24}{25}$ 65. $\pi/3, 5\pi/3$
67. $\pi/4, 3\pi/4, 5\pi/4, 7\pi/4$ 69. $\pi/6, \pi/2, 5\pi/6, 3\pi/2$
71. $0, \pi, 2\pi$ 73. $0 \leq x \leq \pi/6$ and $5\pi/6 \leq x \leq 2\pi$
75. $0 \leq x < \pi/4, 3\pi/4 < x < 5\pi/4, 7\pi/4 < x \leq 2\pi$

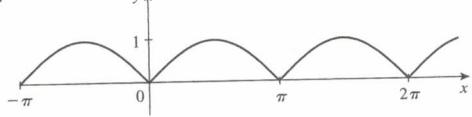
77.



79.



81.

89. 14.34457 cm²

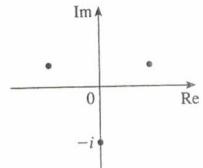
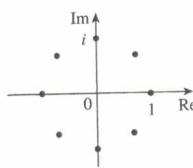
EXERCISES E ■ PAGE A38

1. $\sqrt{1} + \sqrt{2} + \sqrt{3} + \sqrt{4} + \sqrt{5}$ 3. $3^4 + 3^5 + 3^6$
 5. $-1 + \frac{1}{3} + \frac{3}{5} + \frac{5}{7} + \frac{7}{9}$ 7. $1^{10} + 2^{10} + 3^{10} + \dots + n^{10}$
 9. $1 - 1 + 1 - 1 + \dots + (-1)^{n-1}$ 11. $\sum_{i=1}^{10} i$

13. $\sum_{i=1}^{19} \frac{i}{i+1}$ 15. $\sum_{i=1}^n 2i$ 17. $\sum_{i=0}^5 2^i$ 19. $\sum_{i=1}^n x^i$
21. 80 23. 3276 25. 0 27. 61 29. $n(n+1)$
31. $n(n^2 + 6n + 17)/3$ 33. $n(n^2 + 6n + 11)/3$
35. $n(n^3 + 2n^2 - n - 10)/4$
41. (a) n^4 (b) $5^{100} - 1$ (c) $\frac{97}{300}$ (d) $a_n - a_0$
43. $\frac{1}{3}$ 45. 14 49. $2^{n+1} + n^2 + n - 2$

EXERCISES G ■ PAGE A55

1. $8 - 4i$ 3. $13 + 18i$ 5. $12 - 7i$ 7. $\frac{11}{13} + \frac{10}{13}i$
 9. $\frac{1}{2} - \frac{1}{2}i$ 11. $-i$ 13. $5i$ 15. $12 + 5i, 13$
 17. $4i, 4$ 19. $\pm \frac{3}{2}i$ 21. $-1 \pm 2i$
 23. $-\frac{1}{2} \pm (\sqrt{7}/2)i$ 25. $3\sqrt{2} [\cos(3\pi/4) + i \sin(3\pi/4)]$
 27. $5 \{ \cos[\tan^{-1}(\frac{4}{3})] + i \sin[\tan^{-1}(\frac{4}{3})] \}$
 29. $4[\cos(\pi/2) + i \sin(\pi/2)], \cos(-\pi/6) + i \sin(-\pi/6),$
 $\frac{1}{2}[\cos(-\pi/6) + i \sin(-\pi/6)]$
 31. $4\sqrt{2} [\cos(7\pi/12) + i \sin(7\pi/12)],$
 $(2\sqrt{2})[\cos(13\pi/12) + i \sin(13\pi/12)], \frac{1}{4}[\cos(\pi/6) + i \sin(\pi/6)]$
 33. -1024 35. $-512\sqrt{3} + 512i$
 37. $\pm 1, \pm i, (1/\sqrt{2})(\pm 1 \pm i)$ 39. $\pm(\sqrt{3}/2) + \frac{1}{2}i, -i$



41. i 43. $\frac{1}{2} + (\sqrt{3}/2)i$ 45. $-e^2$
 47. $\cos 3\theta = \cos^3 \theta - 3 \cos \theta \sin^2 \theta,$
 $\sin 3\theta = 3 \cos^2 \theta \sin \theta - \sin^3 \theta$